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On the covariant quantization of type-II superstrings

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Abstract: In a series of papers Grassi, Policastro, Porrati and van Nieuwenhuizen have introduced a new method to covariantly quantize the GS-superstring by constructing a resolution of the pure spinor constraint of Berkovits' approach. Their latest version is based on a gauged WZNW model and a definition of physical states in terms of relative cohomology groups. We first put the off-shell formulation of the type-II version of their ideas into a chirally split form and directly construct the free action of the gauged WZNW model, thus circumventing some complications of the super group manifold approach to type-II. Then we discuss the BRST charges that define the relative cohomology and the N=2 superconformal algebra. A surprising result is that nilpotency of the BRST charge requires the introduction of another quartet of ghosts.

Keywords: Superstrings and Heterotic Strings, Conformal Field Models in String Theory, BRST Symmetry.

Contents

1. Introduction

The classical Green Schwarz superstring[1]

$$\mathcal{L}^{GS} = -\frac{1}{2}\Pi_\mu^m \Pi_m^\mu + \mathcal{L}_{WZ} \tag{1.1}$$

$$\Pi_\mu^m = \partial_\mu x^m - i\theta\gamma^m\partial_\mu\theta - i\hat{\theta}\gamma^m\partial_\mu\hat{\theta} \tag{1.2}$$

$$\mathcal{L}_{WZ} = -i\varepsilon^{\mu\nu}\Pi_\mu^m\left((\theta\gamma_m\partial_\nu\theta) - (\hat{\theta}\gamma_m\partial_\nu\hat{\theta})\right) - \varepsilon^{\mu\nu}(\theta\gamma_m\partial_\mu\theta)(\hat{\theta}\gamma^m\partial_\nu\hat{\theta}) \tag{1.3}$$

is covariant and manifestly spacetime supersymmetric. In this last feature it differs from the RNS string, where space time supersymmetry only comes in after the GSO projection. The problem for the Green Schwarz string on the other hand is that a covariant quantization with the standard BRST procedure does not work. The reason for this misery is a set of 16 mixed first and second class constraints $d_{z\alpha}$ that cannot be split easily into first and second class type in a covariant manner. The conjugate momenta $p_{z\alpha}$ of θ^α can be

[1] x^m, θ^α, $\hat{\theta}^{\hat{\alpha}}$ are the coordinates of the target-superspace, where θ^α and $\hat{\theta}^{\hat{\alpha}}$ are Majorana-Weyl, i.e. they each have 16 real components.

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entirely expressed by other phase space variables. $p_{z\alpha}$ minus these expressions are thus constraints in the hamiltonian formalism, namely $d_{z\alpha}$. Half of the constraints are first class and correspond to a fermionic gauge symmetry, known as κ-symmetry. Siegel [1] had the idea to make $d_{z\alpha}$ part of a closed algebra by adding the generators that arise via the Poisson bracket of d with itself. At the quantum level this translates into the current algebra

$$\begin{aligned} id_{z\alpha}(z)id_{z\beta}(w) &\sim -2i\frac{\gamma^m_{\alpha\beta}\Pi_{zm}(w)}{z-w} & (1.4)\\ id_{z\alpha}(z)\Pi_{zm}(w) &\sim -2\frac{\gamma_{m\,\alpha\beta}\partial\theta^\beta(w)}{z-w} & (1.5)\\ \Pi_{zm}(z)\Pi_{zn}(w) &\sim -\frac{\eta_{mn}}{(z-w)^2} & (1.6)\\ id_{z\alpha}(z)\partial\theta^\beta(w) &\sim -\frac{i\delta_\alpha^\beta}{(z-w)^2} & (1.7)\end{aligned}$$

which corresponds to a chiral symmetry algebra of a free action

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2}\partial x^m\bar\partial x_m + p_{z\alpha}\bar\partial\theta^\alpha + \hat p_{\bar z\hat\alpha}\partial\hat\theta^{\hat\alpha} & (1.8)\\ &= -\frac{1}{2}\Pi_z^m\Pi_{\bar z m} + \mathcal{L}_{WZ} + d_{z\alpha}\bar\partial\theta^\alpha + \hat d_{\bar z\hat\alpha}\partial\hat\theta^{\hat\alpha} & (1.9)\\ d_{\mu\alpha} &\equiv p_{\mu\alpha} - (\gamma_m\theta)_\alpha\left(i\partial_\mu x^m + \frac{1}{2}\theta\gamma^m\partial_\mu\theta + \frac{1}{2}\hat\theta\gamma^m\partial_\mu\hat\theta\right) & (1.10)\\ \hat d_{\mu\hat\alpha} &\equiv \hat p_{\mu\hat\alpha} - (\gamma_m\hat\theta)_{\hat\alpha}\left(i\partial_\mu x^m + \frac{1}{2}\theta\gamma^m\partial_\mu\theta + \frac{1}{2}\hat\theta\gamma^m\partial_\mu\hat\theta\right) & (1.11)\end{aligned}$$

It has the interesting feature that it coincides with the Green-Schwarz string, if one imposes the additional constraints $d_{z\alpha} = 0$ and $\hat d_{\bar z\hat\alpha} = 0$. In the free theory, $d_{z\alpha}$ is a priori not a hamiltonian constraint, but the generator of a chiral (not local) symmetry. The second class property of $d_{z\alpha}$ in the Green-Schwarz string is reflected in the free theory by the fact that the OPE of $d_{z\alpha}$ with itself does not form a closed subalgebra. Berkovits [2] had the idea to implement the constraint cohomologically with a BRST operator disregarding its non-closure

$$Q = -\oint \mathrm{d}z i\lambda^\alpha d_{z\alpha} - \oint \mathrm{d}\bar z i\hat\lambda^{\hat\alpha}\hat d_{\bar z\hat\alpha} \qquad (1.12)$$

where λ^α is a commuting ghost. For first class constraints the BRST cohomology can be built, because the BRST operator is nilpotent due to the closure of the algebra. For second class constraints, however, the non-closure implies a lack of nilpotency of the BRST operator. To overcome this problem, Berkovits put a constraint on the ghost field λ and $\hat\lambda$, the so called pure spinor constraint

$$\lambda\gamma^m\lambda = 0 \qquad (1.13)$$

In a series of papers [3]–[9] Grassi, Policastro, Porrati and van Nieuwenhuizen have removed this extra constraint by adding additional ghost variables. They realized in [7] that their theory has the stucture of a gauged WZNW model with the complete diagonal subgroup gauged. It is based on the chiral algebra above (note that we changed the sign of $p_{z\alpha}$ and $d_{z\alpha}$ in our definitions as compared to [7]). A current can be set to zero by gauging the corresponding symmetry and thus making it a first class constraint. However, $d_{z\alpha}$ does not form a subalgebra and thus cannot be gauged on its own. So if one starts gauging $d_{z\alpha}$ and tries to make the resulting BRST-operator (1.12) nilpotent by adding further ghosts, one automatically arrives at a BRST operator that corresponds to a theory where also Π_{zm} and $\partial\theta^\alpha$ are gauged (see e.g. [6, p.7] or [7, p.4]). But this also removes the physical fields from cohomology. Therefore a grading was introduced by hand to get the correct cohomology. In a recent beautiful paper [9] this procedure was replaced by a new approach to the conversion of second class constraints into first class, which was worked out for the gauging of a coset of a (simple) Lie algebra. The idea, which may work for a more general set of constraints that generate a first class algebra, is to gauge the complete algebra and later undo the gauging of the unwanted constraints by building a relative cohomology with respect to a second BRST operator. In the present paper we discuss this program for the type-II string with some modifications.

Despite of the elegance of the group supermanifold approach, there are some puzzling points about the WZNW action:

- For the heterotic string one starts with a chiral algebra and gets from the WZNW model a chiral as well as an antichiral algebra. Somehow one has to get rid of the antichiral one.
- For the type-II string one starts with a chiral and an antichiral algebra. Both of them double and, more severely, because of the central extension of the supersymmetry algebra, the Jacobi identity forces one to introduce an additional generator with two spinor indices. Thus it has not been possible yet for us to produce a WZNW action for the type-II string.

Although we hope that those problems can be overcome, we shortcut the procedure by directly constructing the free field version of the gauged WZNW action.

The paper is organized as follows:

- In section 2 we start with the free field action, discuss its off-shell symmetry algebra generated by the current $d_{z\alpha}$ and gauge it to turn $d_{z\alpha}$ into a constraint. Before actually gauging the algebra via the Noether procedure, we make it close off-shell. To this end we introduce auxiliary fields P_{zm} and $P_{\bar{z}m}$. There still remain double poles in the current algebra, which cause trouble in the gauging procedure. They can be eliminated by doubling all fields, as it was done in [7], in order to establish nilpotent BRST transformations. Gauge fixing leads to the BRST-transformations as they are given in [7].
- In 3 we follow the ideas in [9] to define a relative cohomology, whose purpose will be to undo the gauging of part of the algebra. A nilpotent second BRST operator can be found for the case of arbitrary generators of a first class algebra, but the construction presumably has to be modified for the superstring.

- In 4 we review the operator algebra presented in [7]. It is straightforwardly extended by the new fields that were introduced in the context of the second BRST operator.

- In 5, we have a closer look at the final BRST operator proposed in [7], which includes diffeomorphism invariance by adding a topological ghost quartet. We come to the conclusion that this operator has to be modified via a second quartet of ghost fields in order to become nilpotent. However, we have not found a suitable form of the second BRST operator at this final stage, because its nilpotency is spoiled by additional terms that are required by diffeomorphism invariance. In this situation one can either do without diffeomorphisms (but keep the $N = 2$ algebra), or return to the grading prescription or a modification thereof.

We also want to mention some other recent work that is based on the Berkovits approach [10, 11, 12].

2. Gauging the current algebra

We start with the free field lagrangian[2]

$$\begin{aligned}
\mathcal{L} &\equiv -\frac{1}{2}\partial x^m \bar{\partial} x_m + p_{z\alpha}\bar{\partial}\theta^\alpha + \hat{p}_{\bar{z}\hat{\alpha}}\partial\hat{\theta}^{\hat{\alpha}} &(2.1)\\
&= -\frac{1}{2}\Pi_z^m \Pi_{\bar{z}m} + \mathcal{L}_{WZ} + d_{z\alpha}\bar{\partial}\theta^\alpha + \hat{d}_{\bar{z}\hat{\alpha}}\partial\hat{\theta}^{\hat{\alpha}} &(2.2)\\
&= -\frac{1}{2}\Pi_\mu^m \Pi_m^\mu + \mathcal{L}_{WZ} + P^{\mu\nu} d_{\mu\alpha}\partial_\nu\theta^\alpha + \bar{P}^{\mu\nu}\hat{d}_{\mu\hat{\alpha}}\partial_\nu\hat{\theta}^{\hat{\alpha}} &(2.3)\\
\Pi_\mu^m &\equiv \partial_\mu x^m - i\theta\gamma^m\partial_\mu\theta - i\hat{\theta}\gamma^m\partial_\mu\hat{\theta} &(2.4)\\
d_{\mu\alpha} &\equiv p_{\mu\alpha} - (\gamma_m\theta)_\alpha\left(i\partial_\mu x^m + \frac{1}{2}\theta\gamma^m\partial_\mu\theta + \frac{1}{2}\hat{\theta}\gamma^m\partial_\mu\hat{\theta}\right) &(2.5)\\
\hat{d}_{\mu\hat{\alpha}} &\equiv \hat{p}_{\mu\hat{\alpha}} - (\gamma_m\hat{\theta})_{\hat{\alpha}}\left(i\partial_\mu x^m + \frac{1}{2}\theta\gamma^m\partial_\mu\theta + \frac{1}{2}\hat{\theta}\gamma^m\partial_\mu\hat{\theta}\right) &(2.6)\\
\mathcal{L}_{WZ} &\equiv -i\varepsilon^{\mu\nu}\Pi_\mu^m\left((\theta\gamma_m\partial_\nu\theta) - (\hat{\theta}\gamma_m\partial_\nu\hat{\theta})\right) - \varepsilon^{\mu\nu}(\theta\gamma_m\partial_\mu\theta)(\hat{\theta}\gamma^m\partial_\nu\hat{\theta})\,. &(2.7)
\end{aligned}$$

From the first line (2.1) to the second line (2.2) we reexpressed the lagrangian in terms of supersymmetric objects.[3] In (2.2) it becomes obvious that the free field action turns into

[2]For a detailed description of our conventions, including the definition of the chiral projector $P^{\mu\nu}$, we refer to the appendix.

[3]Supersymmetry acts on the superspace coordinates x^m, θ^α and $\hat{\theta}^{\hat{\alpha}}$ as follows

$$\begin{aligned}
\delta_\varepsilon\theta^\alpha &= \varepsilon^\alpha\\
\delta_\varepsilon\hat{\theta}^{\hat{\alpha}} &= \hat{\varepsilon}^{\hat{\alpha}}\\
\delta_\varepsilon x^m &= i(\varepsilon\gamma^m\theta) + i(\hat{\varepsilon}\gamma^m\hat{\theta})\\
\Rightarrow\quad \delta_\varepsilon\partial\theta^\alpha &= \delta_\varepsilon\partial\hat{\theta}^{\hat{\alpha}} = \delta_\varepsilon\Pi_\mu^m = 0\,.
\end{aligned}$$

The transformations of $p_{\mu\alpha}$ and $\hat{p}_{\mu\hat{\alpha}}$ under supersymmetry δ_ε are defined in such a way that $d_{\mu\alpha}$ is SUSY inert.

$$\delta_\varepsilon p_{\mu\alpha} = (\gamma_m\varepsilon)_\alpha\left(i\partial_\mu x^m + \frac{1}{2}\theta\gamma^m\partial_\mu\theta + \frac{1}{2}\hat{\theta}\gamma^m\partial_\mu\hat{\theta}\right) - \frac{1}{2}(\gamma_m\theta)_\alpha\left(\varepsilon\gamma^m\partial_\mu\theta + \hat{\varepsilon}\gamma^m\partial_\mu\hat{\theta}\right)$$

the Green Schwarz action for the type-II superstring if $d_{z\alpha} = 0$ and $\hat{d}_{\bar{z}\hat{\alpha}} = 0$ are put as extra constraints. The free field equations ($\partial\bar{\partial}x^m = \bar{\partial}\theta^\alpha = \bar{\partial}p_{z\alpha} = \partial\hat{\theta}^{\hat{\alpha}} = \partial\hat{p}_{\bar{z}\hat{\alpha}} = 0$) imply that $d_{z\alpha}$ is a conserved chiral and $\hat{d}_{\bar{z}\hat{\alpha}}$ is a conserved antichiral Noether current

$$\bar{\partial}d_{z\alpha} \stackrel{\text{eom}}{=} 0\,, \qquad \partial\hat{d}_{\bar{z}\hat{\alpha}} \stackrel{\text{eom}}{=} 0\,. \tag{2.8}$$

They thus belong to symmetries of the free field action. As discussed in the introduction, we want to turn those currents into constraints by gauging the symmetries. Instead of the imaginary objects $d_{z\alpha}$ and $\hat{d}_{\bar{z}\alpha}$, we take $id_{z\alpha}$ and $i\hat{d}_{\bar{z}\hat{\alpha}}$ as currents. Although we want to discuss the off-shell symmetries, the symmetry algebra (on-shell) can be most neatly written down in terms of operator products. We will do so, in order to get an idea, how we have to proceed. On the operator level, $id_{z\alpha}$ is part of the following operator algebra:

$$id_{z\alpha}(z)id_{z\beta}(w) \sim -2i\frac{\gamma^m_{\alpha\beta}\Pi_{zm}(w)}{z-w} \tag{2.9}$$

$$id_{z\alpha}(z)\Pi_{zm}(w) \sim -2\frac{\gamma_{m\,\alpha\beta}\partial\theta^\beta(w)}{z-w} \tag{2.10}$$

$$\Pi_{zm}(z)\Pi_{zn}(w) \sim -\frac{\eta_{mn}}{(z-w)^2} \tag{2.11}$$

$$id_{z\alpha}(z)\partial\theta^\beta(w) \sim -\frac{i\delta_\alpha^\beta}{(z-w)^2} \tag{2.12}$$

Here it becomes obvious that d does not form a closed subalgebra. This reflects the fact that it corresponds to second class constraints in the Green Schwarz string. Instead, the remaining currents Π_{zm} and $\partial\theta^\alpha$ form a (centrally extended) subalgebra. Following the instructions of [9], we thus have to gauge the complete algebra and later build the relative cohomology of the BRST operator with respect to a second one which will undo the gauging of Π and $\partial\theta$.

Let us rewrite the algebra in a more general notation:[4]

$$J_{M_1}(z)J_{M_2}(w) \sim -\frac{J_{M_3}f^{M_3}{}_{M_1M_2}}{z-w} - \frac{\mathcal{H}_{M_1M_2}}{(z-w)^2} \tag{2.13}$$

$$\begin{aligned}\delta_\varepsilon\hat{p}_{\mu\hat{\alpha}} &= (\gamma_m\hat{\varepsilon})_{\hat{\alpha}}\left(i\partial_\mu x^m + \frac{1}{2}\theta\gamma^m\partial_\mu\theta + \frac{1}{2}\hat{\theta}\gamma^m\partial_\mu\hat{\theta}\right) - \frac{1}{2}(\gamma_m\hat{\theta})_{\hat{\alpha}}\left(\varepsilon\gamma^m\partial_\mu\theta + \hat{\varepsilon}\gamma^m\partial_\mu\hat{\theta}\right)\\ \Rightarrow\quad \delta_\varepsilon d_{\mu\alpha} &= \delta_\varepsilon\hat{d}_{\mu\hat{\alpha}} = 0\end{aligned}$$

$\mathcal{L}_{WZ}$ is not invariant itself, but transforms into a total divergence.

$$\delta_\varepsilon\int\mathcal{L}_{WZ} = 0\,.$$

We want to have a theory in the end which is manifestly supersymmetric. Therefore the BRST differential to be constructed should anticommute with supersymmetry. This will be a leading thought in the following.

[4]Here we have a different sign in the definition of the structure constants as compared to [7]. The reason is that in our conventions the algebra

$$J_{M_1}(z)J_{M_2}(w) \sim -\frac{J_{M_3}f^{M_3}{}_{M_1M_2}}{z-w} - \frac{\mathcal{H}_{M_1M_2}}{(z-w)^2}$$

corresponds to a Poisson bracket algebra

$$\{J_{M_1}(\sigma^-), J_{M_2}(\sigma'^-)\} = 4\pi J_{M_3}f^{M_3}{}_{M_1M_2}\delta(\sigma^1-\sigma'^1) - 8\pi\mathcal{H}_{M_1M_2}\partial_1\delta(\sigma^1-\sigma'^1)\,.$$

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$$J_{zM} \equiv J_M \equiv (J_m, J_\alpha, J_{\underline{\alpha}}) \equiv (\Pi_{zm}, id_{z\alpha}, \partial\theta^\alpha) \tag{2.14}$$

$$\mathcal{H}_{M_1M_2} \equiv \begin{pmatrix} \eta_{m_1m_2} & 0 & 0 \\ 0 & 0 & \mathcal{H}_{\alpha_1\underline{\alpha}_2} = i\delta_{\alpha_1}^{\alpha_2} \\ 0 & \mathcal{H}_{\underline{\alpha}_1\alpha_2} = -i\delta_{\alpha_2}^{\alpha_1} & 0 \end{pmatrix} \tag{2.15}$$

$$f^{m_3}{}_{\alpha_1\alpha_2} \equiv 2i\gamma^m_{\alpha\beta} = f^{m_3}{}_{\alpha_2\alpha_1} \tag{2.16}$$

$$f^{\underline{\alpha}_3}{}_{\alpha_1 m_2} \equiv 2\gamma_{m_2\,\alpha_1\alpha_3} = -f^{\underline{\alpha}_3}{}_{m_2\alpha_1}\,. \tag{2.17}$$

We can use the metric and its graded inverse

$$\mathcal{H}^{M_1M_2} \equiv \begin{pmatrix} \eta^{m_1m_2} & 0 & 0 \\ 0 & 0 & \mathcal{H}^{\alpha_1\underline{\alpha}_2} = -i\delta_{\alpha_2}^{\alpha_1} \\ 0 & \mathcal{H}^{\underline{\alpha}_1\alpha_2} = i\delta_{\alpha_1}^{\alpha_2} & 0 \end{pmatrix} \tag{2.18}$$

to pull indices:

$$J^M \equiv (J^m, J^\alpha, J^{\underline{\alpha}}) = J_N\mathcal{H}^{NM} = (\Pi_z^m, i\partial\theta^\alpha, d_{z\alpha})\,. \tag{2.19}$$

For the antichiral currents $\hat{J}_{\bar{z}M} \equiv \hat{J}_{\hat{M}} \equiv (\hat{J}_m, \hat{J}_{\hat{\alpha}}, \hat{J}_{\underline{\hat{\alpha}}}) \equiv (\Pi_{\bar{z}m}, i\hat{d}_{\bar{z}\alpha}, \bar{\partial}\hat{\theta}^{\hat{\alpha}})$, we define a metric $\hat{\mathcal{H}}_{\hat{M}\hat{N}}$ and structure constants $\hat{f}^{\hat{K}}{}_{\hat{M}\hat{N}}$ that are numerically equal to those above.

2.1 The form of the transformations to be gauged

Let us also define a generalized notation for the field content we have so far[5]

$$\phi^{\mathcal{A}} \equiv (\phi^m, \phi^\alpha, \phi^{\underline{\alpha}}, \phi^{\hat{\alpha}}, \phi^{\underline{\hat{\alpha}}}) \equiv (x^m, \theta^\alpha, p_{z\alpha}, \hat{\theta}^{\hat{\alpha}}, \hat{p}_{\bar{z}\hat{\alpha}})\,. \tag{2.20}$$

Introduce transformation parameters ω^M and $\hat{\omega}^{\hat{M}}$ with Lie-Algebra indices

$$\omega^M(z) \equiv (\omega^m, \omega^\alpha, \omega^{\underline{\alpha}}) = (\omega^m, \omega^\alpha, -i\omega_\alpha) \tag{2.21}$$

$$\omega_M(z) \equiv (\omega_m, \omega_\alpha, \omega_{\underline{\alpha}}) = (\omega_m, \omega_\alpha, -i\omega^\alpha) \tag{2.22}$$

$$\hat{\omega}^{\hat{M}}(\bar{z}) \equiv (\hat{\omega}^m, \hat{\omega}^{\hat{\alpha}}, \hat{\omega}^{\underline{\hat{\alpha}}}) = (\hat{\omega}^m, \hat{\omega}^{\hat{\alpha}}, -i\hat{\omega}_{\hat{\alpha}}) \tag{2.23}$$

$$\hat{\omega}^{\hat{M}}(\bar{z}) \equiv (\hat{\omega}_m, \hat{\omega}_{\hat{\alpha}}, \hat{\omega}_{\underline{\hat{\alpha}}}) = (\hat{\omega}_m, \hat{\omega}_{\hat{\alpha}}, -i\hat{\omega}^{\hat{\alpha}})\,. \tag{2.24}$$

Noether's theorem tells us that one gets the current by a local variation of the fields

$$\delta_\omega S = \int \frac{\delta S}{\delta\phi^{\mathcal{A}}}\delta_\omega\phi^{\mathcal{A}} \stackrel{!}{=} \int J_M\bar{\partial}\omega^M = -\int \bar{\partial}J_M\omega^M \tag{2.25}$$

with

$$\frac{\delta S}{\delta\phi^{\mathcal{A}}} = (\partial\bar{\partial}x^m, -\bar{\partial}p_{z\alpha}, -\bar{\partial}\theta^\alpha, -\partial\hat{p}_{\bar{z}\hat{\alpha}}, -\partial\hat{\theta}^{\hat{\alpha}})\,. \tag{2.26}$$

This is excatly the classical algebra that one gets from a WZNW-model with level $n = -2$ and with currents

$$J_M \equiv \langle T_M\,,\, g^{-1}\partial g\rangle \quad \Longleftrightarrow \quad g^{-1}\partial g = T_M J^M$$
$$\text{with } [T_{M_1}, T_{M_2}] = T_{M_3}f^{M_3}{}_{M_1M_2}\,.$$

Therefore we think that the sign of the structure constants in [7], eq. (2.3) is not consistent with the choice $[T_{M_1}, T_{M_2}] = T_{M_3}f^{M_3}{}_{M_1M_2}$. That explains the need for a non-invariant metric H_{MN} in [7] to pull the indices of the currents once.

[5]We call those fields "matter fields" in the following, in order to distinguish them from ghost fields and auxiliary fields, which we are going to introduce later.

Given the currents, we thus can read off the transformations according to (2.25)[6]

$$
\begin{aligned}
-\int \bar{\partial} J_M \omega^M &= \int -\bar{\partial}\Pi_{zm}\omega^m - \bar{\partial} i d_{z\alpha}\omega^\alpha + i\bar{\partial}\partial\theta^\alpha\omega_\alpha && (2.27)\\
&= \int \bar{\partial}\partial x_m \Big\{ -\omega^m + (\omega\gamma^m\theta)\Big\} - \bar{\partial} p_{z\alpha}\,(i\omega^\alpha) - \\
&\quad -\bar{\partial}\theta^\alpha \Big\{ i\partial\omega_\alpha - 2i\omega^m(\gamma_m\partial\theta)_\alpha - i\partial\omega^m(\gamma_m\theta)_\alpha + (\gamma_m\omega)_\alpha\partial x^m + \\
&\qquad + \frac{3i}{2}(\omega\gamma_m\theta)(\gamma^m\partial\theta)_\alpha + \frac{i}{2}(\partial\omega\gamma_m\theta)(\gamma^m\theta)_\alpha \Big\} - \\
&\quad -\partial\hat{\theta}^{\hat{\alpha}} \Big\{ -i(\gamma_m\hat{\theta})_\alpha\bar{\partial}\omega^m + \frac{i}{2}(\bar{\partial}\omega\gamma_m\theta)(\gamma^m\hat{\theta})_{\hat{\alpha}} \Big\}\,. && (2.28)
\end{aligned}
$$

There is, however, an arbitrariness: Adding $(\gamma_m\mu)_\alpha(\hat{\theta}\gamma^m\partial\hat{\theta})$ with some parameter μ^α to the bracket behind $\bar{\partial}\theta$ and adding at the same time a term $(\mu\gamma_m\bar{\partial}\theta)(\gamma^m\hat{\theta})_{\hat{\alpha}}$ to the bracket behind $\partial\hat{\theta}^{\hat{\alpha}}$ does not change anything. This is a special form of a *trivial gauge transformation*[7] of $p_{z\alpha}$ and $\hat{p}_{\bar{z}\hat{\alpha}}$. Trivial transformations do not change the currents. We have started with supersymmetric currents and would have expected that they correspond to transformations which commute with supersymmetry. But this is only true up to trivial gauge transformations. We do want transformations that commute with supersymmetry because those transformations will later (after gauging and gauge fixing) turn into BRST transformations. If we want an off-shell formalism which is manifestly supersymmetric, BRST should commute with supersymmetry. We therefore choose the transformations in such a way that this is fulfilled. We further add right now the transformations which correspond to the antiholomorphic currents:[8]

$$
\begin{aligned}
\delta x^m &= -\omega^m + (\omega\gamma^m\theta) - \hat{\omega}^m + (\hat{\omega}\gamma^m\hat{\theta}) && (2.29)\\
\delta\theta^\alpha &= i\omega^\alpha && (2.30)
\end{aligned}
$$

[6]In this and several future calculations, one has to use the Fierz identity $\gamma_{m\,\alpha(\beta}\gamma^m_{\gamma\delta)} = 0$.

[7]A trivial gauge transformation (see e.g. [13, p.69]) is of the form

$$\delta\phi^M = (-)^N A^{MN}\frac{\delta S}{\delta\phi^N}$$

with A^{MN} graded antisymmetric. It is a local symmetry that is present for any theory with more than one field, but it does not imply a gauge freedom:

$$\delta S = \frac{\delta S}{\delta\phi^M}\delta\phi^M = (-)^N\frac{\delta S}{\delta\phi^M}A^{MN}\frac{\delta S}{\delta\phi^N} = 0\,.$$

In the present case we have

$$
\begin{aligned}
A^{\underline{\alpha}\underline{\hat{\beta}}} &= (\gamma_m\hat{\mu})_{\hat{\beta}}(\gamma^m\theta)_\alpha + (\gamma_m\mu)_\alpha(\gamma^m\hat{\theta})_{\hat{\beta}}\\
A^{\underline{\hat{\alpha}}\underline{\beta}} &= (\gamma_m\hat{\mu})_{\hat{\alpha}}(\gamma^m\theta)_\beta + (\gamma_m\mu)_\beta(\gamma^m\hat{\theta})_{\hat{\alpha}}\,.
\end{aligned}
$$

[8]We also could have derived all this using operator product expansions (or classically via the Poisson-bracket). In the quantized theory, the currents are generators of the symmetry transformations via the operator product:

$$\delta_\omega\phi^N(w) = \operatorname*{Res}_{z\to w} J_M(z)\omega^M(z)\phi^N(w)\,.$$

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$$\delta p_{z\alpha} = i\partial\omega_\alpha - 2i\omega^m(\gamma_m\partial\theta)_\alpha - i\partial\omega^m(\gamma_m\theta)_\alpha + +(\gamma_m\omega)_\alpha\partial x^m + \\ +\frac{3i}{2}(\omega\gamma_m\theta)(\gamma^m\partial\theta)_\alpha + \frac{i}{2}(\partial\omega\gamma_m\theta)(\gamma^m\theta)_\alpha - \frac{3i}{2}(\gamma_m\omega)_\alpha(\hat\theta\gamma^m\partial\hat\theta) - \\ -i\partial\hat\omega_m(\gamma^m\theta)_\alpha + \frac{3i}{2}(\hat\omega\gamma_m\partial\hat\theta)(\gamma^m\theta)_\alpha + \frac{i}{2}(\partial\hat\omega\gamma^m\hat\theta)(\gamma_m\theta)_\alpha \tag{2.31}$$

$$\delta\hat\theta^{\hat\alpha} = i\hat\omega^{\hat\alpha} \tag{2.32}$$

$$\delta\hat p_{\bar z\hat\alpha} = i\bar\partial\hat\omega_{\hat\alpha} - 2i\hat\omega_m(\gamma^m\bar\partial\hat\theta)_{\hat\alpha} - i\bar\partial\hat\omega_m\,(\gamma^m\hat\theta)_{\hat\alpha} + (\gamma_m\hat\omega)_{\hat\alpha}\bar\partial x^m + \\ +\frac{3i}{2}(\hat\omega\gamma_m\hat\theta)(\gamma^m\bar\partial\hat\theta)_{\hat\alpha} + \frac{i}{2}(\bar\partial\hat\omega\,\gamma_m\hat\theta)\left(\gamma^m\hat\theta\right)_{\hat\alpha} - \frac{3i}{2}(\gamma_m\hat\omega)_{\hat\alpha}(\theta\gamma^m\bar\partial\theta) - \\ -i\bar\partial\omega^m(\gamma_m\hat\theta)_\alpha + \frac{3i}{2}(\omega\gamma_m\bar\partial\theta)(\gamma^m\hat\theta)_{\hat\alpha} + \frac{i}{2}(\bar\partial\omega\gamma_m\theta)(\gamma^m\hat\theta)_{\hat\alpha}\,. \tag{2.33}$$

It becomes obvious that these transformations commute with supersymmetry, when one varies supersymmetric objects. The results should again be supersymmetric, which is indeed the case:

$$\delta\Pi_{zm} = -\partial\omega_m + 2(\omega\gamma_m\partial\theta) - \partial\hat\omega_m + 2(\hat\omega\gamma_m\partial\hat\theta) \tag{2.34}$$

$$\delta i d_{z\alpha} = -\partial\omega_\alpha + 2i(\gamma_m\omega)_\alpha\Pi_z^m + 2\omega^m(\gamma_m\partial\theta)_\alpha \tag{2.35}$$

$$\delta\partial\theta^\alpha = i\partial\omega^\alpha \tag{2.36}$$

$$\delta\Pi_{\bar z m} = -\bar\partial\omega_m + 2(\omega\gamma_m\bar\partial\theta) - \bar\partial\hat\omega_m + 2(\hat\omega\gamma_m\bar\partial\hat\theta) \tag{2.37}$$

$$\delta i\hat d_{\bar z\hat\alpha} = -\bar\partial\hat\omega_{\hat\alpha} + 2i(\gamma_m\hat\omega)_{\hat\alpha}\Pi_{\bar z}^m + 2\hat\omega^m(\gamma_m\bar\partial\hat\theta)_{\hat\alpha} \tag{2.38}$$

$$\delta\bar\partial\hat\theta^{\hat\alpha} = i\bar\partial\hat\omega^{\hat\alpha}\,. \tag{2.39}$$

In the condensed notation, this can be written as

$$\delta J_M = -\partial\omega_M + J_P f^P{}_{MN}\omega^N + (\hat\omega - \text{terms}) \tag{2.40}$$

$$\delta\hat J_{\hat M} = -\bar\partial\hat\omega_{\hat M} + \hat J_{\hat P}\hat f^{\hat P}{}_{\hat M\hat N}\hat\omega^{\hat N} + (\omega - \text{terms})\,. \tag{2.41}$$

The fact that the transformation of J_M contains $\hat\omega$-terms reflects the non-closure of the off-shell algebra of the transformations.

2.2 Closing the off-shell-algebra

In order to arrive at an off-shell nilpotent BRST differential, one needs an off-shell closed gauge algebra. The commutators of the transformations generated by Π_{zm}, $id_{z\alpha}$, $\partial\theta^\alpha$, $\Pi_{\bar z m}$, $i\hat d_{\bar z\hat\alpha}$ and $\bar\partial\hat\theta^{\hat\alpha}$ generate further transformations which we have not yet included in the

---

The basic OPEs of the free field theory in our conventions are

$$\partial x^m(z)\partial x^n(w) \sim -\frac{\eta^{mn}}{(z-w)^2}$$

$$p_{z\alpha}(z)\theta^\beta(w) \sim \frac{-\delta_\alpha^\beta}{z-w} \sim d_{z\alpha}(z)\theta^\beta(w)$$

However, the resulting transformations would not contain a priori the terms $-i(\gamma_m\hat\theta)_\alpha\bar\partial\omega^m + \frac{i}{2}(\bar\partial\omega\gamma_m\theta)(\gamma^m\hat\theta)_{\hat\alpha}$ in $\delta\hat p_{\bar z\hat\alpha}$ which are necessary to produce the right off-shell currents. In addition, the same considerations about supersymmetry and trivial transformations have to be done there, too.

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algebra. This becomes evident only when a commutator of transformations is acting on d or $\hat{d}$.

$$[\delta_1, \delta_2] d_{z\alpha} = 2\partial\left((\gamma_m\omega_1)_\alpha \omega_2^m - \omega_1^m(\gamma_m\omega_2)_\alpha\right) + 4(\gamma_m\partial\theta)_\alpha(\omega_1\gamma^m\omega_2) + \\ +2(\gamma_m\omega_2)_\alpha\left(-\partial\hat{\omega}_1^m + 2(\hat{\omega}_1\gamma^m\partial\hat{\theta})\right) - 2(\gamma_m\omega_1)_\alpha\left(-\partial\hat{\omega}_2^m + 2(\hat{\omega}_2\gamma^m\partial\hat{\theta})\right). \tag{2.42}$$

The first two terms correspond to transformations with parameter ω_α and ω^m respectively. The last two terms show the non-closure of the algebra. Furthermore, $2(\gamma_m\omega_2)_\alpha\partial\hat{\omega}_1^m$ and $2(\gamma_m\omega_1)_\alpha\partial\hat{\omega}_2^m$ correspond to pure shifts in $d_{z\alpha}$ or equivalently in $p_{z\alpha}$. The holomorphic current for this transformation is θ^α which is not supersymmetric and thus would spoil commutativity of BRST and SUSY. The reason for these terms to show up, is the Π_{zm} in the transformation of $d_{z\alpha}$ (2.35). The transformation of Π_{zm} (2.34) contains (in contrast to the heterotic case) also hatted variables.

In [5, p.11], a nilpotent BRST transformation of $d_{z\alpha}$ was achieved by introducing an auxiliary variable P_0^m and rewriting the transformations in terms of ∂_1-derivatives only. We use a similar ansatz to get a closed off-shell algebra (this is of course closely related to BRST-nilpotency). However, to keep the manifest chiral split, we introduce two such variables.

The symmetry algebra closes as long as we consider only the transformations with parameter ω^M or only transformations with parameter $\hat{\omega}^{\hat{M}}$. This would correspond to the transformations in a chiral or antichiral heterotic case. In order to get rid of the terms in question, one can thus introduce auxiliary fields P_z and $P_{\bar{z}}$ in the transformation of d_z and $\hat{d}_{\bar{z}\hat{\alpha}}$ that transform as Π_z and $\Pi_{\bar{z}}$ do in the chiral or antichiral heterotic case respectively:

$$\tilde{\delta} d_{z\alpha} = i\partial\omega_\alpha + 2(\gamma_m\omega)_\alpha P_z^m - 2i\omega^m(\gamma_m\partial\theta)_\alpha \tag{2.43}$$

$$= \delta d_{z\alpha} - 2(\gamma_m\omega)_\alpha\left(\Pi_z^m - P_z^m\right) \tag{2.44}$$

$$\tilde{\delta} P_z^m \overset{!}{=} -\partial\omega^m + 2(\omega\gamma^m\partial\theta)\,. \tag{2.45}$$

Equivalently for $\hat{d}_{\bar{z}\hat{\alpha}}$ and $P_{\bar{z}m}$. These transformations are still supersymmetric within type-II if P is defined to be SUSY-inert. At the same time we have to guarantee that the equations of motions of the matter fields do not change, that on shell P coincides with Π and that the invariance of the action under the "global" transformations (holomorphic ω^M and antiholomorphic $\hat{\omega}^M$) is conserved. This can be achieved by adding a term proportional to $(P - \Pi)^2$ to the action[9]

$$\tilde{\mathcal{L}} = \mathcal{L} + \frac{c}{2}\left(P_z^m - \Pi_z^m\right)\left(P_{\bar{z}m} - \Pi_{\bar{z}m}\right) \tag{2.46}$$

with some parameter c yet to be determined by the invariance condition. All the other transformations remain unchanged.[10] The variation of the action under local transforma-

[9]Varying with respect to P_z^m and $P_{\bar{z}}^m$ yields for $c \neq 0$ algebraic equations of motion $P_z^m = \Pi_z^m$ and $P_{\bar{z}}^m = \Pi_{\bar{z}}^m$. Reinserting those equations returns the old action. For $c = 1$ the Π^2-term cancels with that of the original action and one gets a first order action.

[10]The change of the transformation of d and $\hat{d}$ has to be implemented by appropriate changes in the transformation of p and $\hat{p}$. However, the information of δp is completely encoded in the transformation of the other elementary fields and the transformation of d. In the following, we will therefore only write down transformations of d instead of p.

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tions reads:

$$
\begin{aligned}
\tilde{\delta}\tilde{S} &= \int \tilde{\delta}L + \frac{c}{2}\cdot\tilde{\delta}\left\{\left(P_z^m - \Pi_z^m\right)\left(P_{\bar{z}m} - \Pi_{\bar{z}m}\right)\right\} && (2.47)\\
&= \int \delta L + (\tilde{\delta}-\delta)d_{z\alpha}\,\bar{\partial}\theta^\alpha + (\tilde{\delta}-\delta)\hat{d}_{\bar{z}\hat{\alpha}}\bar{\partial}\hat{\theta}^{\hat{\alpha}} + && (2.48)\\
&\quad + c\cdot\left\{\left((\hat{\omega}\gamma^m\partial\hat{\theta}) - \frac{1}{2}\partial\hat{\omega}^m\right)\left(\Pi_{\bar{z}m} - P_{\bar{z}m}\right) + \left((\omega\gamma^m\bar{\partial}\theta) - \frac{1}{2}\bar{\partial}\omega^m\right)\left(\Pi_{zm} - P_{zm}\right)\right\}\\
&= \int \delta L + \frac{c}{2}\bar{\partial}\omega^m\left(P_{mz} - \Pi_{zm}\right) + \frac{c}{2}\partial\hat{\omega}^m\left(P_{\bar{z}m} - \Pi_{\bar{z}m}\right) + && (2.49)\\
&\quad + (-2+c)\,(\omega\gamma^m\bar{\partial}\theta)\left(\Pi_z^m - P_z^m\right) + (-2+c)\,(\hat{\omega}\gamma^m\partial\hat{\theta})\left(\Pi_{\bar{z}}^m - P_{\bar{z}}^m\right).
\end{aligned}
$$

For the global variation to vanish, we thus have to choose

$$
c = 2\,. \tag{2.50}
$$

The local variation shows that the old currents J_m and $\hat{J}_m$ change by $(P_{mz} - \Pi_{zm})$ and $(P_{\bar{z}m} - \Pi_{\bar{z}m})$ respectively.

The action is now of the form (we drop the tilde$\tilde{}$of the new action and transformations)

$$
\begin{aligned}
S &= \int P_z^m P_{\bar{z}m} - P_z^m \Pi_{\bar{z}m} - \Pi_z^m P_{\bar{z}m} + \frac{1}{2}\Pi_z^m \Pi_{\bar{z}m} + \mathcal{L}_{WZ} + d_{z\alpha}\bar{\partial}\theta^\alpha + \hat{d}_{\bar{z}\hat{\alpha}}\partial\hat{\theta}^{\hat{\alpha}} && (2.51)\\
&= \int \left(P_z^m - \Pi_z^m\right)\left(P_{\bar{z}m} - \Pi_{\bar{z}m}\right) - \frac{1}{2}\partial x^m \bar{\partial} x_m + p_{z\alpha}\bar{\partial}\theta^\alpha + \hat{p}_{\bar{z}\hat{\alpha}}\bar{\partial}\hat{\theta}^{\hat{\alpha}} && (2.52)
\end{aligned}
$$

and is invariant under global chiral and antichiral transformations. The new local transformations read:

$$
\begin{aligned}
\delta d_{z\alpha} &= i\partial\omega_\alpha + 2(\gamma_m\omega)_\alpha P_z^m - 2i\omega^m(\gamma_m\partial\theta)_\alpha && (2.53)\\
\delta\hat{d}_{\bar{z}\hat{\alpha}} &= i\bar{\partial}\hat{\omega}_{\hat{\alpha}} + 2(\gamma_m\hat{\omega})_{\hat{\alpha}} P_{\bar{z}}^m - 2i\hat{\omega}^m(\gamma_m\bar{\partial}\hat{\theta})_{\hat{\alpha}} && (2.54)\\
\delta P_z^m &= -\partial\omega^m + 2(\omega\gamma^m\partial\theta) && (2.55)\\
\delta P_{\bar{z}}^m &= -\bar{\partial}\hat{\omega}^m + 2(\hat{\omega}\gamma^m\bar{\partial}\hat{\theta})\,. && (2.56)
\end{aligned}
$$

The off-shell algebra for the local transformations is now closed.

2.3 Gauging

Gauging an algebra means introducing some gauge fields (connections) such that the corresponding symmetry becomes local. Therefore one gets a gauge symmetry with its generators (the currents) constrained to zero. This can be done by an iterative procedure known under the name Noether's method (see e.g. [14, 15]).

From the local variation (2.50) we have seen that the only currents that change off-shell are $\Pi_{zm} \to P_{zm}$ and $\Pi_{\bar{z}m} \to P_{\bar{z}m}$. The new currents are thus

$$
\begin{aligned}
J_{zM} &= \left(P_{zm}, id_{z\alpha}, \partial\theta^\alpha\right) && (2.57)\\
\hat{J}_{\bar{z}\hat{M}} &= \left(P_{\bar{z}m}, i\hat{d}_{\bar{z}\hat{\alpha}}, \bar{\partial}\hat{\theta}^{\hat{\alpha}}\right). && (2.58)
\end{aligned}
$$

They transform in the following way

$$\delta J_M = -\partial\omega_M + J_P f^P{}_{MN}\omega^N \tag{2.59}$$
$$\delta \hat{J}_{\hat{M}} = -\bar{\partial}\hat{\omega}_{\hat{M}} + \hat{J}_{\hat{P}}\hat{f}^{\hat{P}}{}_{\hat{M}\hat{N}}\hat{\omega}^{\hat{N}}\,. \tag{2.60}$$

Now we are going to perfom the *Noether procedure.* For simplicity we will stay in the condensed notation and only consider the chiral symmetry. The antichiral symmetry is treated analogously. We are going to introduce gauge fields $A_{\bar{z}}^M$ in order to make the symmetries local. It is useful for bookkeeping to define a grading that counts the number of gauge fields. This induces a grading for the variation δ, depending on whether it increases or decreases the grading of the fields. We will denote the grading by a lower index. So far we have

$$S_0 = \int P_z^m P_{\bar{z}m} - P_z^m \Pi_{\bar{z}m} - \Pi_z^m P_{\bar{z}m} + \frac{1}{2}\Pi_z^m \Pi_{\bar{z}m} + \mathcal{L}_{WZ} + d_{z\alpha}\bar{\partial}\theta^\alpha + \hat{d}_{\bar{z}\hat{\alpha}}\partial\hat{\theta}^{\hat{\alpha}} \tag{2.61}$$
$$\delta_0 S_0 = \int J_{zM}\bar{\partial}\omega^M\,. \tag{2.62}$$

The next well known step is to add a coupling of gauge fields to the currents

$$S_1 = -\int J_{zM}A_{\bar{z}}^M\,. \tag{2.63}$$

Defining

$$\delta_{-1}A_{\bar{z}}^M = \bar{\partial}\omega^M \tag{2.64}$$

the variation of order zero vanishes ($\delta_{-1}J_M = 0$ as J_M does not contain any gauge fields and thus cannot be decreased in their number)

$$\delta_0 S_0 + \delta_{-1}S_1 = 0\,. \tag{2.65}$$

Terms of order 1 in the grading are $\delta_1 S_0$, $\delta_0 S_1$ and $\delta_{-1}S_2$. We would be done, if $\delta_0 S_1$vanished on its own.

$$\delta_0 S_1 = \int -\delta_0 J_{zM}\,A_{\bar{z}}^M - J_{zM}\delta_0 A_{\bar{z}}^M \tag{2.66}$$
$$= \int -\left(J_{zP}f^P{}_{MN}\omega^N - \partial\omega_M\right)A_{\bar{z}}^M - J_{zP}\delta_0 A_{\bar{z}}^P\,. \tag{2.67}$$

We can only cancel the first term by defining

$$\delta_0 A_{\bar{z}}^P = -f^P{}_{MN}\omega^N A_{\bar{z}}^M \tag{2.68}$$
$$\Rightarrow \quad \delta_0 S_1 = \int \partial\omega_M A_{\bar{z}}^M\,. \tag{2.69}$$

The terms $-\partial\omega_M$ in the transformation (2.59) of the currents keep the procedure from terminating after this first step. These terms correspond to the double poles in the current algebra (2.13). One can now use a simple trick and introduce a number of auxiliary fields that produce the same algebra with a different sign for the double poles. Adding those currents to the original ones makes the double poles vanish. The simplest way to do this, is to *double the fields* (including P), and subtract from the original lagrangian the same lagrangian in terms of the auxiliary fields.[11] This is similar to the gauged WZNW model, where one arrives at an analogous doubling after gauge fixing and an adequate parametrization of the surviving gauge fields [16, 17]. The original variables in WZNW are embedding functions g into a group manifold, and the auxiliary fields are usually called h. The coordinates of the free theory do not parametrize a group manifold (since $p_{z\alpha}$ is an elementary field). But because of this similarity, we put an index h to our auxiliary variables and refer to the resulting currents as h-currents.

$$S \equiv S_{\text{old}} - S_h \,. \tag{2.70}$$

The h-action is of course seperately invariant under the same chiral transformations expressed in h-coordinates as the original lagrangian. Let us call the corresponding transformation parameters ω^h_M. For our purpose we have to choose $\omega^h_M = \omega_M$. In addition to (2.29), (2.30), (2.32) and (2.53)–(2.56) we get the following transformations:

$$\begin{aligned}
\delta x^{h\,m} &= -\omega^m + (\omega\gamma^m\theta^h) - \hat{\omega}^m + (\hat{\omega}\gamma^m\theta^h) && (2.71)\\
\delta\theta^{h\,\alpha} &= i\omega^\alpha && (2.72)\\
\delta d^h_{z\alpha} &= i\partial\omega_\alpha + 2(\gamma_m\omega)_\alpha P^m_z - 2i\omega^m(\gamma_m\partial\theta)_\alpha && (2.73)\\
\delta P^{h\,m}_z &= -\partial\omega^m + 2(\omega\gamma^m\partial\theta) && (2.74)\\
\delta\hat{\theta}^{h\,\hat{\alpha}} &= i\hat{\omega}^{\hat{\alpha}} && (2.75)\\
\delta\hat{d}^h_{\bar{z}\hat{\alpha}} &= i\partial\hat{\omega}_{\hat{\alpha}} + 2(\gamma_m\hat{\omega})_\alpha P^{h\,m}_{\bar{z}} - 2i\hat{\omega}^m(\gamma_m\partial\hat{\theta}^h)_{\hat{\alpha}} && (2.76)\\
\delta P^{h\,m}_{\bar{z}} &= -\bar{\partial}\hat{\omega}^m + 2(\hat{\omega}\gamma^m\bar{\partial}\hat{\theta}^h)\,. && (2.77)
\end{aligned}$$

Variation of the action under local transformations yields

$$\delta_\omega S = \int \left(J_M + J^h_M\right)\bar{\partial}\omega^M + \left(\hat{J}_{\hat{M}} + \hat{J}^h_{\hat{M}}\right)\partial\hat{\omega}^{\hat{M}} \tag{2.78}$$

$$\text{where } J^h_M \equiv -(P^h_{zm}, id_{z\alpha}, \partial\theta^\alpha) \qquad \hat{J}^h_{\hat{M}} \equiv -(P^h_{\bar{z}m}, i\hat{d}_{\bar{z}\hat{\alpha}}, \bar{\partial}\theta^{\hat{\alpha}})\,. \tag{2.79}$$

The transformation of the complete currents now has vanishing double poles (no $\partial\omega^M$-terms)

$$\begin{aligned}
\delta(J_{zM} + J^h_{zM}) &= (J_{zP} + J^h_{zP})f^P{}_{MN}\omega^N && (2.80)\\
\delta(\hat{J}_{\bar{z}\hat{M}} + \hat{J}^h_{\bar{z}\hat{M}}) &= (\hat{J}_{\bar{z}P} + \hat{J}^h_{\bar{z}P})\hat{f}^{\hat{P}}{}_{\hat{M}\hat{N}}\omega^{\hat{N}} && (2.81)
\end{aligned}$$

and the gauged action only needs the coupling to the connection

$$S_{\text{gauged}} = S - S_h - \int (J_M + J^h_M)A^M_{\bar{z}} - \int (\hat{J}_{\hat{M}} + \hat{J}^h_{\hat{M}})\hat{A}^{\hat{M}}_{\bar{z}} \tag{2.82}$$

with

$$\begin{aligned}
\delta A^P_{\bar{z}} &= \bar{\partial}\omega^P - f^P{}_{MN}\omega^N A^M_{\bar{z}} && (2.83)\\
\delta\hat{A}^{\hat{P}}_z &= \partial\hat{\omega}^{\hat{P}} - \hat{f}^{\hat{P}}{}_{\hat{M}\hat{N}}\hat{\omega}^{\hat{N}}\hat{A}^{\hat{M}}_z\,. && (2.84)
\end{aligned}$$

[11] In [7, p.8] this trick was used to make the BRST transformations nilpotent.

2.4 Gauge fixing and BRST transformation

One can use the gauge freedom to put all the gauge fields to zero again. Within the standard BRST formalism we introduce ghosts by rewriting the transformation parameters as

$$\omega^M \equiv \Lambda c^M \,, \qquad \hat{\omega}^{\hat{M}} \equiv \Lambda \hat{c}^{\hat{M}} \tag{2.85}$$

with an anticommuting imaginary global parameter Λ and the ghost fields c^M. The BRST differential s on the elementary fields is then defined as the transformation without this parameter

$$\delta_\omega \phi^M \equiv \Lambda \cdot s\phi^M \,. \tag{2.86}$$

The action is of course still invariant under this transformation. We add the usual gauge fixing and ghost term to the lagrangian:

$$\mathcal{L}_{\text{qu}} = \mathcal{L}_{\text{gauged}} \; + s(b_M A_{\bar{z}}^M) \tag{2.87}$$

with

$$sb_M = \Lambda_M \tag{2.88}$$

$$s\Lambda_M = 0 \tag{2.89}$$

where Λ_M is the Lagrange multiplier field and has nothing to do with the scalar parameter Λ. The BRST-transformation of the ghosts is defined such that s becomes nilpotent.

From the gauge transformation of the gauge field

$$\delta A_{\bar{z}}^P = \bar{\partial}\omega^P - f^P{}_{MN}\omega^N A_{\bar{z}}^M \tag{2.90}$$

we can read off its BRST transformation by pulling the parameter Λ to the front:[12]

$$sA_{\bar{z}}^P = \bar{\partial}c^P - (-)^{N+M+P} f^P{}_{MN} c^N A_{\bar{z}}^M \tag{2.91}$$

$$\Rightarrow \quad s(b_P A_{\bar{z}}^P) = \Lambda_P A_{\bar{z}}^P - (-)^P b_P \bar{\partial} c^P + (-)^{N+M} b_P f^P{}_{MN} c^N A_{\bar{z}}^M \,. \tag{2.92}$$

The complete quantum action is thus

$$S_{\text{qu}} = S_{\text{gauged}} + \int \Lambda_P A_{\bar{z}}^P - (-)^P b_P \bar{\partial} c^P + (-)^{N+M} b_P f^P{}_{MN} c^N A_{\bar{z}}^M \,. \tag{2.93}$$

Varying with respect to Λ_M and $A_{\bar{z}}^M$ yields algebraic equations for those fields

$$A_{\bar{z}}^M = 0 \tag{2.94}$$

$$\Lambda_M = (J_M + J_M^h) - (-)^{N+M} b_P f^P{}_{MN} c^N \tag{2.95}$$

[12]The ghosts as well as the antighosts have an anticommuting body, i.e. the grading of c^M is

$$| c^M | = | M | + 1 \,.$$

It thus makes sense to have contractions of the form $(-)^P b_P c^P$.

which can be reinserted to finally arrive at the following action (which we will call simply S again):

$$\begin{aligned} S = \int & P_z^m P_{\bar{z}m} - P_z^m \Pi_{\bar{z}m} - \Pi_z^m P_{\bar{z}m} + \frac{1}{2}\Pi_z^m \Pi_{\bar{z}m} + \mathcal{L}_{WZ} + d_{z\alpha}\bar{\partial}\theta^\alpha + \hat{d}_{\bar{z}\hat{\alpha}}\partial\hat{\theta}^{\hat{\alpha}} - \\ & - \Big(P_z^{h\,m} P_{\bar{z}m}^h - P_z^{h\,m}\Pi_{\bar{z}m}^h - \Pi_z^{h\,m} P_{\bar{z}m}^h + \frac{1}{2}\Pi_z^{h\,m}\Pi_{\bar{z}m}^h + \mathcal{L}_{WZ}^h + d_{z\alpha}^h \bar{\partial}\theta^{h\,\alpha} + \hat{d}_{\bar{z}\hat{\alpha}}^h \partial\hat{\theta}^{h\,\hat{\alpha}} \Big) + \\ & + \beta_{zm}\bar{\partial}\xi^m + \omega_{z\alpha}\bar{\partial}\lambda^\alpha + \kappa_z^\alpha \bar{\partial}\chi_\alpha + \hat{\beta}_{\bar{z}m}\partial\hat{\xi}^m + \hat{\omega}_{\bar{z}\hat{\alpha}}\bar{\partial}\hat{\lambda}^{\hat{\alpha}} + \hat{\kappa}_{\bar{z}}^{\hat{\alpha}}\bar{\partial}\hat{\chi}_{\hat{\alpha}} \,. \end{aligned} \tag{2.96}$$

Here we have defined[13]

$$c^M \equiv (-\xi^m, \lambda^\alpha, \chi_\alpha) \tag{2.97}$$

$$\Rightarrow \quad c_M = (-\xi_m, i\chi_\alpha, -i\lambda^\alpha) \tag{2.98}$$

and

$$b_M \equiv (\beta_{zm}, \omega_{z\alpha}, \kappa_z^\alpha) \tag{2.99}$$

$$\Rightarrow \quad b^M = (\beta_z^m, i\kappa_z^\alpha, -i\omega_{z\alpha}) \tag{2.100}$$

and equivalent relations for the hatted ghosts. From the former gauge transormations (2.29), (2.30), (2.32), (2.53)–(2.56) and (2.71)–(2.77) we can read off the BRST transformations

$$sx^m = \xi^m + (\lambda\gamma^m\theta) + \hat{\xi}^m + (\hat{\lambda}\gamma^m\hat{\theta}) \tag{2.101}$$

$$s\theta^\alpha = i\lambda^\alpha \tag{2.102}$$

$$sd_{z\alpha} = -\partial\chi_\alpha + 2(\gamma_m\lambda)_\alpha P_z^m + 2i\xi^m(\gamma_m\partial\theta)_\alpha \tag{2.103}$$

$$sP_z^m = \partial\xi^m + 2(\lambda\gamma^m\partial\theta) \tag{2.104}$$

$$s\hat{\theta}^{\hat{\alpha}} = i\hat{\lambda}^{\hat{\alpha}} \tag{2.105}$$

$$s\hat{d}_{\bar{z}\hat{\alpha}} = -\bar{\partial}\hat{\chi}_{\hat{\alpha}} + 2(\gamma_m\hat{\lambda})_{\hat{\alpha}} P_{\bar{z}}^m + 2i\hat{\xi}^m(\gamma_m\bar{\partial}\hat{\theta})_{\hat{\alpha}} \tag{2.106}$$

$$sP_{\bar{z}}^m = \bar{\partial}\hat{\xi}^m + 2(\hat{\lambda}\gamma^m\bar{\partial}\hat{\theta}) \tag{2.107}$$

$$sx^{h\,m} = \xi^m + (\lambda\gamma^m\theta^h) + \hat{\xi}^m + (\hat{\lambda}\gamma^m\hat{\theta}^h) \tag{2.108}$$

$$\vdots$$

The transformation of the supersymmetric momentum has the form

$$s\Pi_\mu^m = \partial_\mu\xi^m + 2(\lambda\gamma^m\partial_\mu\theta) + \partial_\mu\hat{\xi}^m + 2(\hat{\lambda}\gamma^m\partial_\mu\hat{\theta}) \,. \tag{2.109}$$

Up to the signs of $p_{z\alpha}$, $d_{z\alpha}$, β_{zm}, $\omega_{z\alpha}$ and κ_z^α these transformations coincide in the chiral sector after integrating out P_{zm} and $P_{\bar{z}m}$ with those in [7].

[13] Like for $p_{z\alpha}$ we use a sign for the antighosts in the action that differs from that in [7].

In order to make the BRST transformations nilpotent, the ghosts have to transform in the following way:[14]

$$sc^M = -(-)^K \frac{1}{2} f^M{}_{LK} c^K c^L \tag{2.110}$$

or in detail

$$s\xi^m = -i(\lambda\gamma^m\lambda) \tag{2.111}$$
$$s\lambda^\alpha = 0 \tag{2.112}$$
$$s\chi_\alpha = 2(\gamma_m\lambda)_\alpha \xi^m \tag{2.113}$$

and similar transformations for the hatted ghosts. The transformation of the antighosts $sb_M = \Lambda_M$ turns via (2.95) into

$$sb_M = (J_M + J^h_M) - (-)^{N+M} b_P f^P{}_{MN} c^N \tag{2.114}$$

or in detail

$$s\beta_{zm} = (P_{zm} - P^h_{zm}) - 2(\kappa_z\gamma_m\lambda) \tag{2.115}$$
$$s\omega_{z\alpha} = (id_{z\alpha} - id^h_{z\alpha}) - 2i\beta_{zm}(\gamma^m\lambda)_\alpha - 2(\gamma_m\kappa_z)_\alpha\xi^m \tag{2.116}$$
$$s\kappa^\alpha_z = (\partial\theta^\alpha - \partial\theta^{h\,\alpha})\,. \tag{2.117}$$

2.5 BRST current, composite B-field and energy momentum tensor

The BRST current can be derived by making the anticommuting transformation parameter Λ of the BRST transformation $\delta_\Lambda(\ldots) = \Lambda s(\ldots)$ local. From the chiral transformations we know that the variation of the "matter part" (without ghosts) is $\int (J_M + J^h_M)\bar\partial(\Lambda c^M)$. The variation of the complete action under local BRST is thus

$$\delta_\Lambda S = \int (J_M+J^h_M)\bar\partial(\Lambda c^M)+(\hat J_{\hat M}+\hat J^h_{\hat M})\partial(\Lambda\hat c^{\hat M})-\delta_\Lambda\Big((-)^M b_M\bar\partial c^M+(-)^{\hat M}\hat b_{\hat M}\bar\partial\hat c^{\hat M}\Big) \tag{2.118}$$
$$= \int \bar\partial\Lambda\Big((-)^M(J_M+J^h_M)c^M+b_M sc^M\Big)+\partial\Lambda\Big((-)^{\hat M}(\hat J_{\hat M}+\hat J^h_{\hat M})\hat c^{\hat M}+\hat b_{\hat M} s\hat c^{\hat M}\Big)\,. \tag{2.119}$$

We can read off the *BRST currents*

$$j^B_z = (-)^M(J_M + J^h_M)c^M - (-)^K\frac{1}{2} b_M f^M{}_{LK} c^K c^L \tag{2.120}$$
$$= -(P_{zm} - P^h_{zm})\xi^m - (id_{z\alpha} - id^h_{z\alpha})\lambda^\alpha - (\partial\theta^\alpha - \partial\theta^{h\,\alpha})\chi_\alpha + $$
$$+i\beta_{zm}(\lambda\gamma^m\lambda) + 2(\kappa_z\gamma_m\lambda)\xi^m \tag{2.121}$$
$$\hat j^B_{\bar z} = -(P_{\bar z m} - P^h_{\bar z m})\hat\xi^m - (i\hat d_{\bar z\hat\alpha} - i\hat d^h_{\bar z\hat\alpha})\hat\lambda^{\hat\alpha} - (\bar\partial\hat\theta^{\hat\alpha} - \bar\partial\hat\theta^{h\,\hat\alpha})\hat\chi_{\hat\alpha} +$$

[14] Taking into accout the appropriate grading signs modifies the general form of the gauge transformations to the following BRST transformations of the currents

$$sJ_N = (-)^{N+M} J_P f^P{}_{NM} c^M - \partial c_N\,.$$

Demanding nilpotency for this formulae in condensed notation yields (using the Jacobi-identity) the transformation of the ghosts.

$$+i\hat{\beta}_{\bar{z}m}(\hat{\lambda}\gamma^m\hat{\lambda}) + 2(\hat{\kappa}_{\bar{z}}\gamma_m\hat{\lambda})\hat{\xi}^m \tag{2.122}$$

which on-shell coincide with that of the chiral or antichiral heterotic string respectively.

For gauged WZNW models there exists in general an operator B_{zz} which makes T_{zz} BRST-exact (see [18, p.24])

$$T_{zz} = [Q, B_{zz}] \tag{2.123}$$

$$\text{where } Q = \oint j^{\mathrm{B}} \,. \tag{2.124}$$

For the covariant superstring based on the gauged WZNW model, it is explicitely written down in [7, p.11] and is important to build up the $N = 2$ superconformal algebra together with the BRST-current, T_{zz} and the ghost current. Having carefully performed the local BRST variation above makes it now simple for us to recognize the symmetry corresponding to B_{zz} in these calculations: Looking at the ghost action

$$S_{gh} = \int -(-)^M b_M\bar{\partial}c^M - (-)^{\hat{M}}\hat{b}_{\hat{M}}\partial\hat{c}^{\hat{M}} = \int -(-)^M c_M\bar{\partial}b^M - (-)^{\hat{M}}\hat{c}_{\hat{M}}\partial\hat{b}^{\hat{M}} \tag{2.125}$$

it becomes clear that b_M and c^M can interchange their role, as long as the conformal weight of b_M is of no importance. That means one can construct a new symmetry by interchanging $c^M \leftrightarrow b^M$ and $\hat{c}^{\hat{M}} \leftrightarrow \hat{b}^{\hat{M}}$, or in detail

$$\begin{aligned} -\xi^m &\leftrightarrow \beta_z^m \\ \lambda^\alpha &\leftrightarrow i\kappa_z^\alpha \\ \chi_\alpha &\leftrightarrow -i\omega_{z\alpha} \end{aligned} \qquad \text{and} \qquad \begin{aligned} -\hat{\xi}^m &\leftrightarrow \hat{\beta}_{\bar{z}}^m \\ \hat{\lambda}^{\hat{\alpha}} &\leftrightarrow i\hat{\kappa}_{\bar{z}}^{\hat{\alpha}} \\ \hat{\chi}_{\hat{\alpha}} &\leftrightarrow -i\hat{\omega}_{\bar{z}\hat{\alpha}} \end{aligned} \tag{2.126}$$

Performing this exchange in all BRST transformations would yield another fermionic nilpotent transformation. However, the aim is that the generator fullfills (2.123). T_{zz} is basically the square of the original currents minus the square of the h-currents plus ghost terms. In the BRST-current, we have $(J + J^h)$-terms. We therefore need $(J - J^h)$ terms in the B_{zz}-current. Changing the relative sign of the transformation parameter for the original fields and for the h-fields does not affect the invariance of the matter action, as the h-part and the original part are invariant independently. The resulting contribution to the current from the matter part is then the difference of J_M and J_M^h, as desired. Call Λ_B the transformation parameter corresponding to B_{zz} and t the fermionic transformation without this parameter

$$\delta_{\Lambda_B}(\ldots) \equiv \Lambda_B t(\ldots)\,. \tag{2.127}$$

The variation of the complete action with respect to this transformation yields

$$\begin{aligned} \delta_{\Lambda_B}S = \int & \bar{\partial}\Lambda_B\left((-)^M(J_M - J_M^h)b^M + c_M t b^M\right) + (-)^M\Lambda_B(J_M - J_M^h)\bar{\partial}b^M - \\ & -(-)^M\Lambda_B t c_M\bar{\partial}b^M + \Lambda_B c_M\bar{\partial}t b^M + \partial\Lambda_B\left((-)^{\hat{M}}(\hat{J}_{\hat{M}} - \hat{J}_{\hat{M}}^h)\hat{b}^{\hat{M}} + \hat{c}_{\hat{M}}t\hat{b}^{\hat{M}}\right) + \\ & +(-)^{\hat{M}}\Lambda_B(\hat{J}_{\hat{M}} - \hat{J}_{\hat{M}}^h)\partial\hat{b}^{\hat{M}} - (-)^{\hat{M}}\Lambda_B t\hat{c}_{\hat{M}}\bar{\partial}\hat{b}^{\hat{M}} + \Lambda_B\hat{c}_{\hat{M}}\bar{\partial}t\hat{b}^{\hat{M}}\,. \end{aligned} \tag{2.128}$$

One can easily complete this transformation to a global symmetry of the whole action by defining $tc_M = (J_M - J^h_M)$ and $tb^M = 0$ and the same for the hatted ghosts. Nilpotency (which is the reason not to make this simple choice in the case of BRST symmetry) is lost already after changing the relative sign between the currents. The holomorphic current for this new symmetry can then be read off to be $(-)^M(J_M - J^h_M)b^M$. In order to obtain the proper energy momentum tensor in an OPE with j^B, this current has to be multiplied with an additional factor of $-1/2$:

$$B_{zz} = -\frac{1}{2}(P_{zm} + P^h_{zm})\beta^m + \frac{i}{2}(id_{z\alpha} + id^h_{z\alpha})\kappa^\alpha_z - \frac{i}{2}(\partial\theta^\alpha + \partial\theta^{h\,\alpha})\omega_{z\alpha} \tag{2.129}$$

$$B_{\bar{z}\bar{z}} = -\frac{1}{2}(P_{\bar{z}m} + P^h_{\bar{z}m})\hat{\beta}^m + \frac{i}{2}(i\hat{d}_{\bar{z}\hat{\alpha}} + i\hat{d}^h_{\bar{z}\hat{\alpha}})\hat{\kappa}^{\hat{\alpha}}_{\bar{z}} - \frac{i}{2}(\bar{\partial}\hat{\theta}^{\hat{\alpha}} + \bar{\partial}\hat{\theta}^{h\,\hat{\alpha}})\hat{\omega}_{z\hat{\alpha}}\,. \tag{2.130}$$

The so defined B_{zz} is a homotopy for the *energy momentum tensor* T_{zz} only on the operator level and not as an off shell current:[15]

$$\begin{aligned}
sB_{zz} \quad &= \quad -(-)^M\frac{1}{2}\Big((J_P - J^h_P)f^P{}_{NM}c^M - 2\partial c_N\Big)\,b^N - \\
&\quad\; -(-)^M\frac{1}{2}(J^M - J^{h\,M})\Big((J_M + J^h_M) - (-)^{N+M}b_P f^P{}_{MN}c^N\Big) &(2.131)\\
&= \quad -\frac{1}{2}J_M J^M + \frac{1}{2}J^h_M J^{h\,M} - (-)^M b_M\partial c^M &(2.132)\\
&= \quad -\frac{1}{2}P_{zm}P^m_z + d_{z\alpha}\partial\theta^\alpha + \frac{1}{2}P^h_{zm}P^{h\,m}_z - d^h_{z\alpha}\partial\theta^{h\,\alpha} + \\
&\quad\; +\beta_{zm}\partial\xi^m + \omega_{z\alpha}\partial\lambda^\alpha + \kappa^\alpha_z\partial\chi_\alpha &(2.133)\\
&\overset{\text{on shell}}{=} T_{zz}\,. &(2.134)
\end{aligned}$$

The off-shell holomorphic energy momentum tensor instead includes Π_z and the hatted fields:

$$\begin{aligned}
T_{zz} = {} & (P_z - \Pi_z)^2 - \frac{1}{2}\Pi_{zm}\Pi^m_z + d_{z\alpha}\partial\theta^\alpha + \hat{d}_{z\hat{\alpha}}\partial\hat{\theta}^{\hat{\alpha}} - \left(P^h_z - \Pi^h_z\right)^2 + \\
& +\frac{1}{2}\Pi^h_{zm}\Pi^{h\,m}_z - d^h_{z\alpha}\partial\theta^{h\,\alpha} - \hat{d}^h_{z\hat{\alpha}}\partial\hat{\theta}^{h\,\hat{\alpha}} + \beta_{zm}\partial\xi^m + \omega_{z\alpha}\partial\lambda^\alpha + \\
& +\kappa^\alpha_z\partial\chi_\alpha + \hat{\beta}_{zm}\partial\hat{\xi}^m + \hat{\omega}_{z\hat{\alpha}}\partial\hat{\lambda}^{\hat{\alpha}} + \hat{\kappa}^{\hat{\alpha}}_z\partial\hat{\chi}_{\hat{\alpha}}\,.
\end{aligned} \tag{2.135}$$

Similarly we have on shell $sB_{\bar{z}\bar{z}} = T_{\bar{z}\bar{z}}$.

3. The second BRST operator

Following the ideas of [9] we now introduce a second BRST-operator and define a relative cohomology whose purpose is to undo the gauging of Π_z, $\partial\theta$, $\Pi_{\bar{z}}$ and $\bar{\partial}\hat{\theta}$. We review the essential ideas and extend the construction to an arbitrary set of constraints that generate a first class system.

[15]Attention: For $J_N - J^h_N$, the double poles do not vanish!

$$s(J_N - J^h_N) = (-)^{N+M}(J_P - J^h_P)f^P{}_{NM}c^M - 2\partial c_N\,.$$

Consider a gauge algebra with generators $G_M = \oint J_M$

$$[G_M, G_N] = G_K f^K{}_{MN} \tag{3.1}$$

and assume that we only want to gauge symmetries which correspond to some subset of generators G_α that do not generate a subalgebra. Call the remaining generators $G_\mathfrak{a}$

$$G_M \equiv (G_\mathfrak{a}, G_\alpha)\,. \tag{3.2}$$

In order to gauge the generators G_α one has to gauge at first the complete algebra and end up at the usual BRST operator $Q = \oint j^{\mathrm{B}}$ of the form (compare (2.120))

$$Q = \oint (-)^M J_M c^M - (-)^K \frac{1}{2} b_M f^M{}_{LK} c^K c^L\,. \tag{3.3}$$

The indices of the ghosts and antighosts split in the same way as those of the generators G_M in (3.2). In order to undo the gauging of $G_\mathfrak{a}$, we set the corresponding ghosts $c^\mathfrak{a}$ cohomologically to zero by making them exact. However, $c^\mathfrak{a}$ cannot be exact with respect to Q, because the BRST transformation of $c^\mathfrak{a}$ is already fixed to something nonzero. Therefore a second BRST operator $Q_c = \oint j_c$ and some new fields have to be introduced. The old ghosts $c^\mathfrak{a}$ and antighosts $b_\mathfrak{a}$, as well as all the new fields will be removed from cohomology via the following diagram:

$$\begin{array}{ccccc} b_\mathfrak{a} & & & & c^\mathfrak{a} \\ & \overset{Q_c}{\searrow} & & \overset{Q_c}{\nearrow} & \\ \downarrow K & & \pi_\mathfrak{a}\ \varphi^\mathfrak{a} & & \uparrow -K \\ & \overset{Q}{\nearrow} & & \overset{Q}{\searrow} & \\ b'_\mathfrak{a} & & & & c'^\mathfrak{a} \end{array} \tag{3.4}$$

Here $c'^\mathfrak{a}$ and $b'_\mathfrak{a}$ are new ghosts and antighosts with grading $\mid \mathfrak{a} \mid +1$, while $\pi_\mathfrak{a}$ and $\varphi^\mathfrak{a}$ are fields with ghost number 0 and grading $\mid \mathfrak{a} \mid$. K will be explained below. The contribution of the new fields to the lagrangian is

$$\mathcal{L}' = -(-)^\mathfrak{a} b'_\mathfrak{a} \bar\partial c'^\mathfrak{a} + \pi_\mathfrak{a} \bar\partial \varphi^\mathfrak{a}\,. \tag{3.5}$$

To comply with (3.4) Q has to be extended by a term $(-)^\mathfrak{a} \pi_\mathfrak{a} c'^\mathfrak{a}$.

$$Q = \oint (-)^M J_M c^M - (-)^K \frac{1}{2} b_M f^M{}_{LK} c^K c^L + (-)^\mathfrak{a} \pi_\mathfrak{a} c'^\mathfrak{a} \tag{3.6}$$

$$\Rightarrow \quad s b'_\mathfrak{a} = \pi_\mathfrak{a} \tag{3.7}$$

$$s\varphi^\mathfrak{a} = c'^\mathfrak{a} \tag{3.8}$$

One can construct a suitable Q_c that anticommutes with Q as the commutator of Q with a homotopy operator K

$$K \equiv \oint k \equiv \oint (-)^\mathfrak{a} b'_\mathfrak{a} c^\mathfrak{a} \tag{3.9}$$

$$\Rightarrow \quad \delta_k c'^\mathfrak{a} = -c^\mathfrak{a} \tag{3.10}$$

$$\delta_k b_{\mathfrak{a}} = b'_{\mathfrak{a}} \tag{3.11}$$

$$Q_c \equiv \oint j_c \equiv [Q, K] \tag{3.12}$$

$$= \oint (-)^{\mathfrak{a}} \pi_{\mathfrak{a}} c^{\mathfrak{a}} + (-)^N \frac{1}{2} b'_{\mathfrak{a}} f^{\mathfrak{a}}{}_{MN} c^N c^M \tag{3.13}$$

with corresponding transformations

$$s_c b_M = \underbrace{s\delta_k b_M}_{\pi_{\mathfrak{a}} \text{ for } M=\mathfrak{a}} + (-)^{N+M} \left(b'_{\mathfrak{b}} f^{\mathfrak{b}}{}_{MN} c^N \right) \tag{3.14}$$

$$s_c \varphi^{\mathfrak{a}} = c^{\mathfrak{a}} \tag{3.15}$$

$$s_c c'^{\mathfrak{a}} = (-)^K \frac{1}{2} f^{\mathfrak{a}}{}_{LK} c^K c^L = -s c^{\mathfrak{a}} \,. \tag{3.16}$$

The Jacobi identity implies

$$[Q, Q_c] = 0 \quad \text{and} \quad [Q_c, Q_c] = [Q, [K, Q_c]] \,. \tag{3.17}$$

It is thus sufficient to check $\delta_K Q_c = 0$, which is obviously the case, to obtain nilpotency of the second BRST operator

$$[Q_c, Q_c] = 0 \,. \tag{3.18}$$

Equation (3.14) differs a little bit from the diagram (3.4). That does not hurt, as $\pi_{\mathfrak{a}}$ according to (3.7) is already BRST exact with respect to Q. Physical observables are then defined to lie in the relative cohomology of Q with respect to Q_c.

Part of Q turns out to be exact with respect to Q_c. If we define

$$\Xi \equiv (-)^{\mathfrak{a}} b_{\mathfrak{a}} c'^{\mathfrak{a}} + \left(J_{\mathfrak{a}} - (-)^{\gamma+\mathfrak{a}} b_\alpha f^\alpha{}_{\mathfrak{a}\gamma} c^\gamma \right) \varphi^{\mathfrak{a}} \tag{3.19}$$

then Q can be written as

$$\begin{aligned} Q = \oint & (-)^\alpha J_\alpha c^\alpha - (-)^\beta \frac{1}{2} b_\gamma f^\gamma{}_{\alpha\beta} c^\beta c^\alpha + s_c \Xi - (-)^{\mathfrak{b}} \frac{1}{2} b_\gamma f^\gamma{}_{\mathfrak{a}\mathfrak{b}} c^{\mathfrak{b}} c^{\mathfrak{a}} - \\ & - (-)^N b'_{\mathfrak{b}} f^{\mathfrak{b}}{}_{\mathfrak{a}N} c^N c'^{\mathfrak{a}} - (-)^{\gamma+\mathfrak{a}+N+\alpha} b'_{\mathfrak{b}} f^{\mathfrak{b}}{}_{\alpha N} c^N f^\alpha{}_{\mathfrak{a}\gamma} c^\gamma \varphi^{\mathfrak{a}} \,. \end{aligned} \tag{3.20}$$

For the gauging of the roots of a simple Lie algebra all structure constants in the second line vanish [9]. For *superstrings*, on the other hand

$$Q = \oint (-)^\alpha J_\alpha c^\alpha + s_c \, \Xi - (-)^\alpha b'_{\mathfrak{b}} f^{\mathfrak{b}}{}_{\mathfrak{a}\alpha} c^\alpha c'^{\mathfrak{a}} \tag{3.21}$$

for the chiral sector. It may therefore be necessary to modify Q or/and K in order to get the Berkovits BRST charge up to Q_c-exact terms.

Actually, for the string there are two algebras with currents

$$J_M = (J_m, J_\alpha, J_{\underline{\alpha}}) = (\Pi_{zm}, id_{z\alpha}, \partial\theta^\alpha) \qquad \hat{J}_{\hat{M}} = (\hat{J}_{\hat{m}}, \hat{J}_{\hat{\alpha}}, \hat{J}_{\underline{\hat{\alpha}}}) = (\Pi_{\bar{z}m}, i\hat{d}_{\bar{z}\hat{\alpha}}, \bar{\partial}\hat{\theta}^{\hat{\alpha}}) \tag{3.22}$$

$$J_{\mathfrak{a}} = (J_m, J_{\underline{\alpha}}) = (\Pi_{zm}, \partial\theta^\alpha) \qquad \hat{J}_{\hat{\mathfrak{a}}} = (\hat{J}_{\hat{m}}, \hat{J}_{\underline{\hat{\alpha}}}) = (\Pi_{\bar{z}m}, \bar{\partial}\hat{\theta}^{\hat{\alpha}}) \,. \tag{3.23}$$

The detailed translation of the ghosts and antighosts and the new fields looks as follows

$$\begin{aligned}
c^{\mathfrak{a}} &\equiv (c^m, c^{\underline{\alpha}}) = (-\xi^m, \chi_\alpha) & \hat{c}^{\hat{\mathfrak{a}}} &\equiv (\hat{c}^m, \hat{c}^{\underline{\hat{\alpha}}}) = (-\hat{\xi}^m, \hat{\chi}_{\hat{\alpha}}) \\
b_{\mathfrak{a}} &\equiv (b_m, c_{\underline{\alpha}}) = (\beta_{zm}, \kappa_z^\alpha) & \hat{b}_{\hat{\mathfrak{a}}} &\equiv (\hat{b}_m, \hat{c}_{\underline{\hat{\alpha}}}) = (\hat{\beta}_{zm}, \hat{\kappa}_z^{\hat{\alpha}}) \\
c'^{\mathfrak{a}} &\equiv (c'^m, c'^{\underline{\alpha}}) = (-\xi'^m, \chi'_\alpha) & \hat{c}'^{\hat{\mathfrak{a}}} &\equiv (\hat{c}'^m, \hat{c}'^{\underline{\hat{\alpha}}}) = (-\hat{\xi}'^m, \hat{\chi}'_{\hat{\alpha}}) \\
b'_{\mathfrak{a}} &\equiv (b'_m, c'_{\underline{\alpha}}) = (\beta'_{zm}, \kappa'^\alpha_z) & \hat{b}'_{\hat{\mathfrak{a}}} &\equiv (\hat{b}'_m, \hat{c}'_{\underline{\hat{\alpha}}}) = (\hat{\beta}'_{zm}, \hat{\kappa}'^{\hat{\alpha}}_z) \\
\varphi^{\mathfrak{a}} &\equiv (\varphi^m, \varphi^{\underline{\alpha}}) \equiv (\varphi^m, -i\varphi_\alpha) & \hat{\pi}_{\hat{\mathfrak{a}}} &\equiv (\hat{\pi}_m, \hat{\pi}_{\underline{\hat{\alpha}}}) \equiv (\hat{\pi}_m, -i\hat{\pi}^{\hat{\alpha}}) \\
\pi_{\mathfrak{a}} &\equiv (\pi_m, \pi_{\underline{\alpha}}) \equiv (\pi_m, -i\pi^\alpha) & \hat{\pi}_{\hat{\mathfrak{a}}} &\equiv (\hat{\pi}_m, \hat{\pi}_{\underline{\hat{\alpha}}}) \equiv (\hat{\pi}_m, -i\hat{\pi}^{\hat{\alpha}})\,.
\end{aligned} \tag{3.24}$$

The general equations thus translate into

$$\begin{aligned}
\mathcal{L}' &= \beta'_{zm}\bar{\partial}\xi'^m + \omega'_{z\alpha}\bar{\partial}\lambda'^\alpha + \kappa'^\alpha_z\bar{\partial}\chi'_\alpha + \pi_{\mathfrak{a}}\partial\varphi^{\mathfrak{a}} + \\
&\quad + \hat{\beta}'_{zm}\bar{\partial}\hat{\xi}'^m + \hat{\omega}'_{z\hat{\alpha}}\bar{\partial}\lambda'^{\hat{\alpha}} + \hat{\kappa}'^{\hat{\alpha}}_z\bar{\partial}\hat{\chi}'_{\hat{\alpha}} + \hat{\pi}_{\hat{\mathfrak{a}}}\partial\hat{\varphi}^{\hat{\mathfrak{a}}} && (3.25)\\
j_z^{\mathrm{B}} &= -(P_{zm} - P^h_{zm})\xi^m - (id_{z\alpha} - id^h_{z\alpha})\lambda^\alpha - (\partial\theta^\alpha - \partial\theta^{h\,\alpha})\chi_\alpha + \\
&\quad + i\beta_{zm}(\lambda\gamma^m\lambda) + 2(\kappa_z\gamma_m\lambda)\xi^m - \pi_m\xi'^m - \pi_{\underline{\alpha}}\chi'_\alpha && (3.26)\\
s\beta'_{zm} &= \pi_m && (3.27)\\
s\kappa'^\alpha_z &= \pi_{\underline{\alpha}} = -i\pi^\alpha && (3.28)\\
s\varphi^m &= -\xi'^m && (3.29)\\
s\varphi^{\underline{\alpha}} &= -is\varphi_\alpha = \chi'_\alpha && (3.30)\\
k_z &\equiv -\beta'_{zm}\xi^m - \kappa'^\alpha_z\chi_\alpha && (3.31)\\
\delta_k\xi'^m &= -\xi^m && (3.32)\\
\delta_k\chi'_\alpha &= -\chi_\alpha && (3.33)\\
\delta_k\beta_{zm} &= \beta'_{zm} && (3.34)\\
\delta_k\kappa^\alpha_z &= \kappa'^\alpha_z && (3.35)\\
j_{cz} &\equiv sk_z = -\pi_{zm}\xi^m - \pi_{z\underline{\alpha}}\chi_\alpha - i\beta'_{zm}(\lambda\gamma^m\lambda) - 2(\kappa'_z\gamma_m\lambda)\xi^m && (3.36)\\
s_c\beta_{zm} &= \pi_m + 2(\kappa'\gamma_m\lambda) && (3.37)\\
s_c\kappa^\alpha_z &= \pi_{\underline{\alpha}} && (3.38)\\
s_c\omega_{z\alpha} &= 2i\beta_{zm}(\gamma^m\lambda)_\alpha + 2(\gamma_m\kappa_z)_\alpha\xi^m && (3.39)\\
s_c\varphi^m &= -\xi^m && (3.40)\\
s_c\varphi^{\underline{\alpha}} &= \chi_\alpha && (3.41)\\
s_c\xi'^m &= -i(\lambda\gamma^m\lambda) = s\xi^m && (3.42)\\
s_c\chi'_\alpha &= 2(\gamma_m\lambda)_\alpha\xi^m = s\chi_\alpha && (3.43)
\end{aligned}$$

and the same for the hatted fields, which are contained in the $\bar{z}$-component of the currents.

The new fields contribute to the on-shell energy momentum tensor in the following way

$$\begin{aligned}
T_{zz} &\to T_{zz} - (-)^{\mathfrak{a}} b'_{\mathfrak{a}}\partial c'^{\mathfrak{a}} + \pi_{\mathfrak{a}}\partial\varphi^{\mathfrak{a}} && (3.44)\\
&= T_{zz} + \beta'_{zm}\partial\xi'^m + \kappa'^\alpha_z\partial\chi'_\alpha + \pi_{z\mathfrak{a}}\partial\varphi^{\mathfrak{a}}\,. && (3.45)
\end{aligned}$$

Changing the composite B-field

$$B_{zz} \to B_{zz} + b'_{\mathfrak{a}} \partial \varphi^{\mathfrak{a}} \tag{3.46}$$

repairs the on-shell relation

$$T_{zz} \overset{\text{on-shell}}{=} sB_{zz} \,. \tag{3.47}$$

4. Operator algebra

Left moving and rightmoving sector do not mix on-shell, and we thus concentrate on the left-moving sector only. On the operator level, we have the same algebra as in [7], namely a Kazama algebra. The new ghosts and fields $c'^{\mathfrak{a}}, b'_{\mathfrak{a}}, \varphi^{\mathfrak{a}}$ and $\pi_{\mathfrak{a}}$ do not disturb this structure, but change the ghost number anomaly. For completeness, we present the algebra here again:[16]

$$\begin{aligned}
T(z)T(w) &\sim \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} && (4.1)\\
T(z)j^{\mathrm{B}}(w) &\sim \frac{j^{\mathrm{B}}(w)}{(z-w)^2} + \frac{\partial j^{\mathrm{B}}(w)}{z-w} && (4.2)\\
T(z)B(w) &\sim \frac{2B(w)}{(z-w)^2} + \frac{\partial B(w)}{z-w} && (4.3)\\
T(z)j^{\mathrm{gh}}(w) &\sim \frac{28}{(z-w)^3} + \frac{j^{\mathrm{gh}}(w)}{(z-w)^2} + \frac{\partial j^{\mathrm{gh}}(w)}{z-w} = \frac{28}{(z-w)^3} + \frac{j^{\mathrm{gh}}(z)}{(z-w)^2} && (4.4)\\
j^{\mathrm{B}}(z)B(w) &\sim \frac{-28}{(z-w)^3} + \frac{j^{\mathrm{gh}}(w)}{(z-w)^2} + \frac{T(w)}{z-w} && (4.5)\\
j^{\mathrm{gh}}(z)j^{\mathrm{B}}(w) &\sim \frac{j^{\mathrm{B}}(w)}{z-w} && (4.6)\\
j^{\mathrm{gh}}(z)B(w) &\sim \frac{-B(w)}{z-w} && (4.7)\\
j^{\mathrm{gh}}(z)j^{\mathrm{gh}}(w) &\sim \frac{-28}{(z-w)^2} && (4.8)\\
B(z)B(w) &\sim \frac{F(w)}{z-w} && (4.9)\\
j^{\mathrm{B}}(z)\Phi(w) &\sim \frac{F(w)}{z-w} && (4.10)
\end{aligned}$$

with

$$\begin{aligned}
T_{zz} &= -\frac{1}{2}\Pi_{zm}\Pi_z^m + d_{z\alpha}\partial\theta^\alpha + \frac{1}{2}\Pi^h_{zm}\Pi_z^{h\,m} - d^h_{z\alpha}\partial\theta^{h\,\alpha} + \\
&\quad + \beta_{zm}\partial\xi^m + \omega_{z\alpha}\partial\lambda^\alpha + \kappa_z^\alpha\partial\chi_\alpha + \beta'_{zm}\partial\xi'^m + \kappa_z'^\alpha\partial\chi'_\alpha + \pi_{z\mathfrak{a}}\partial\varphi^{\mathfrak{a}} && (4.11)\\
j_z^{\mathrm{B}} &= -(\Pi_{zm} - \Pi^h_{zm})\xi^m - (id_{z\alpha} - id^h_{z\alpha})\lambda^\alpha - (\partial\theta^\alpha - \partial\theta^{h\,\alpha})\chi_\alpha + \\
&\quad + i\beta_{zm}(\lambda\gamma^m\lambda) + 2(\kappa_z\gamma_m\lambda)\xi^m - \pi_m\xi'^m - \pi_{\underline{\alpha}}\chi'_\alpha && (4.12)\\
B_{zz} &= -\frac{1}{2}(\Pi_{zm} + \Pi^h_{zm})\beta^m + \frac{i}{2}(id_{z\alpha} + id^h_{z\alpha})\kappa_z^\alpha - \frac{i}{2}(\partial\theta^\alpha + \partial\theta^{h\,\alpha})\omega_{z\alpha} + b'_{\mathfrak{a}}\partial\varphi^{\mathfrak{a}} && (4.13)
\end{aligned}$$

[16] We used the OPE package OPE-defs.m by K. Thielemans [19].

$$j_z^{\rm gh} = \beta_{zm}\xi^m + \omega_{z\alpha}\lambda^\alpha + \kappa_z^\alpha\chi_\alpha + \beta'_{zm}\xi'^m + \kappa_z'^\alpha\chi'_\alpha \tag{4.14}$$

$$F_{zzz} \equiv -i\beta_{zm}\left(\kappa_z\gamma^m(\partial\theta - \partial\theta^h)\right) + \frac{i}{2}(\kappa_z\gamma^m\kappa_z)\left(\Pi_{zm} - \Pi^h_{zm}\right) \tag{4.15}$$

$$\Phi_{zzz} \equiv \frac{i}{2}\beta_{zm}(\kappa_z\gamma^m\kappa_z)\,. \tag{4.16}$$

The only term that prevents the algebra from coinciding with an $N = 2$ superconformal algebra is the BRST-exact operator F. To turn the Kazama algebra into an $N = 2$ superconformal algebra, B can be made nilpotent by adding a quartet of two anticommuting (b'_{zz}, c'^z) and two commuting ghosts (β'_{zz}, γ'^z) as it is done in [7].[17] The ghosts form a quartet

$$sc'^z \equiv -\gamma'^z \tag{4.17}$$

$$s\beta'_{zz} \equiv -b'_{zz} \tag{4.18}$$

and thus do not contribute to the cohomology. Having an additional term $b'_{zz}\bar\partial c'^z + \beta'_{zz}\bar\partial\gamma'^z$ in the lagrangian, this corresponds to the following new term in the BRST-current:

$$j^{\rm B} \to j^{\rm B} + b'_{zz}\gamma'^z\,. \tag{4.19}$$

The term F in the algebra disappears when one changes B to

$$B_{zz} \to B_{zz} - 2\beta'_{zz}\partial c'^z - c'^z\partial\beta'_{zz} - b'_{zz} - \frac{1}{2}c'^z F_{zzz} - \frac{1}{2}\gamma'^z\Phi_{zzz}\,. \tag{4.20}$$

The energy momentum tensor now reads

$$\begin{aligned} T_{zz} = & -\frac{1}{2}\Pi_{zm}\Pi^m_z + d_{z\alpha}\partial\theta^\alpha + \frac{1}{2}\Pi^h_{zm}\Pi^{h\,m}_z - d^h_{z\alpha}\partial\theta^{h\,\alpha} + \beta_{zm}\partial\xi^m + \omega_{z\alpha}\partial\lambda^\alpha + \kappa_z^\alpha\partial\chi_\alpha + \\ & + \beta'_{zm}\partial\xi'^m + \kappa_z'^\alpha\partial\chi'_\alpha + \pi_{z\mathfrak{a}}\partial\varphi^{\mathfrak{a}} + 2\beta'_{zz}\partial\gamma'^z + \partial\beta'_{zz}\gamma'^z + 2b'_{zz}\partial c'^z + \partial b'_{zz}c'^z \end{aligned} \tag{4.21}$$

and the ghost current becomes

$$j_z^{\rm gh} = \beta_{zm}\xi^m + \omega_{z\alpha}\lambda^\alpha + \kappa_z^\alpha\chi_\alpha + \beta'_{zm}\xi'^m + \kappa_z'^\alpha\chi'_\alpha + b'_{zz}c'^z + 2\beta'_{zz}\gamma'^z\,. \tag{4.22}$$

The resulting current algebra is a twisted $N = 2$ superconformal algebra, as was already shown in [7]:

$$T(z)T(w) \sim \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} \tag{4.23}$$

$$T(z)j^{\rm B}(w) \sim \frac{j^{\rm B}(w)}{(z-w)^2} + \frac{\partial j^{\rm B}(w)}{z-w} \tag{4.24}$$

[17] In [7] this quartet is called $(b_{zz}, c^z, \beta_{zz}, \gamma^z)$, and b_{zz} and c^z are later identified with the worldsheet diffeomorphism ghosts. However, we disagree with the final BRST charge $\mathbf{Q}$ of [7], eqs. (6.1)–(6.3), because it is not nilpotent. In particular, the operator Q_V defined in (6.1) does not square to zero. It might be that the resulting terms are Q_c exact, which would be enough to build a relative cohomology, but we decided to seperate the steps of making B_{zz} nilpotent via a first quartet $(b'_{zz}, c'^z, \beta'_{zz}, \gamma'^z)$ and of including worldsheet diffeomorphism invariance via a second quartet $(b_{zz}, c^z, \beta_{zz}, \gamma^z)$.

$$T(z)B(w) \sim \frac{2B(w)}{(z-w)^2} + \frac{\partial B(w)}{z-w} \tag{4.25}$$

$$T(z)j^{\rm gh}(w) \sim \frac{31}{(z-w)^3} + \frac{j^{\rm gh}(w)}{(z-w)^2} + \frac{\partial j^{\rm gh}(w)}{z-w} = \frac{31}{(z-w)^3} + \frac{j^{\rm gh}(z)}{(z-w)^2} \tag{4.26}$$

$$j^{\rm B}(z)B(w) \sim \frac{-31}{(z-w)^3} + \frac{j^{\rm gh}(w)}{(z-w)^2} + \frac{T(w)}{z-w} \tag{4.27}$$

$$j^{\rm gh}(z)j^{\rm B}(w) \sim \frac{j^{\rm B}(w)}{z-w} \tag{4.28}$$

$$j^{\rm gh}(z)B(w) \sim \frac{-B(w)}{z-w} \tag{4.29}$$

$$j^{\rm gh}(z)j^{\rm gh}(w) \sim \frac{-31}{(z-w)^2} \tag{4.30}$$

$$B(z)B(w) \sim 0\,. \tag{4.31}$$

The usual structure can be seen by defining operators $\check{T} \equiv T - \frac{1}{2}\partial j^{\rm gh}$, $J \equiv j^{\rm gh}$, $G^+ \equiv j^{\rm B}$,$G^- \equiv 2B$. This is basically a topological twist in T. The central charge of the superconformal algebra becomes $c = -93$:

$$\check{T}(z)\check{T}(w) \sim \frac{c/2}{(z-w)^4} + \frac{2\check{T}(w)}{(z-w)^2} + \frac{\partial\check{T}(w)}{z-w} \tag{4.32}$$

$$\check{T}(z)G^{\pm} \sim \frac{3/2\cdot G^{\pm}(w)}{(z-w)^2} + \frac{\partial G^{\pm}(w)}{z-w} \tag{4.33}$$

$$\check{T}(z)J(w) \sim \frac{J(w)}{(z-w)^2} + \frac{\partial J(w)}{z-w} = \frac{J(z)}{(z-w)^2} \tag{4.34}$$

$$G^+(z)G^-(w) \sim \frac{2c/3}{(z-w)^3} + \frac{2J(w)}{(z-w)^2} + \frac{2\check{T}(w) + \partial J(w)}{z-w} \tag{4.35}$$

$$J(z)G^{\pm}(w) \sim \frac{\pm J(w)}{z-w} \tag{4.36}$$

$$J(z)J(w) \sim \frac{c/3}{(z-w)^2}\,. \tag{4.37}$$

5. Worldsheet diffeomorphism invariance

In order to implement worldsheet diffeomorphism invariance, we would have to gauge the symmetry corresponding to T_{zz}. The algebra (especially (4.27)) tells us that we then also have to gauge B_{zz}.[18] We have not explicitely performed this gauging, but after gauge fixing, one expects the additional terms $c^z(T_{zz} + \frac{1}{2}T^{\rm top}_{zz}) + \gamma^z(B_{zz} + \frac{1}{2}B^{\rm top}_{zz})$ as given in [20, p.125]. This was of course the idea in [7], but there the final BRST operator was not nilpotent as mentioned in footnote 17. We now add the topological quartet $(b_{zz}, c^z, \beta_{zz}, \gamma^z)$. This quartet itself obeys an $N = 2$ superconformal algebra, if one defines

$$T^{\rm top}_{zz} = 2\beta_{zz}\partial\gamma^z + \partial\beta_{zz}\gamma^z + 2b_{zz}\partial c^z + \partial b_{zz}c^z \tag{5.1}$$

[18] In the twisted theory, this would mean gauging one of the supersymmetry generators and thus establishing $N = 1$ SUGRA.

$$
\begin{aligned}
j_z^{\mathrm{B,top}} &= b_{zz}\gamma^z && (5.2)\\
B_{zz}^{\mathrm{top}} &= -2\partial c^z\,\beta_{zz} - c^z\partial\beta_{zz} && (5.3)\\
j^{\mathrm{gh,top}} &= b_{zz}c^z + 2\beta_{zz}\gamma^z\,. && (5.4)
\end{aligned}
$$

The ghosts decouple from the cohomology because

$$
\begin{aligned}
sc^z &\equiv -\gamma^z && (5.5)\\
s\beta_{zz} &\equiv -b_{zz}\,. && (5.6)
\end{aligned}
$$

And the lagrangian is of the form

$$
\mathcal{L}^{\mathrm{top}} = b_{zz}\bar{\partial}c^z + \beta_{zz}\bar{\partial}\gamma^z\,. \tag{5.7}
$$

We want to keep T BRST-exact, and therefore we need $j_z^{\mathrm{B,top}}$ as part of our final BRST current:

$$
\begin{aligned}
&T_{zz} = [Q, B_{zz}] && (5.8)\\
&T_{zz}^{\mathrm{top}} = [Q^{\mathrm{top}}, B_{zz}^{\mathrm{top}}] && (5.9)\\
\Rightarrow\ &\left[Q+Q^{\mathrm{top}}+\oint c^z\left(T_{zz}+\frac{1}{2}T_{zz}^{\mathrm{top}}\right)+\gamma^z\left(B_{zz}+\frac{1}{2}B_{zz}^{\mathrm{top}}\right),\, B_{zz}+B_{zz}^{\mathrm{top}}\right] = T_{zz}+T_{zz}^{\mathrm{top}}\,. && (5.10)
\end{aligned}
$$

The final BRST current now reads

$$
\begin{aligned}
\mathbf{j}_{\mathbf{z}}^{\mathrm{B}} = &-(\Pi_{zm}-\Pi^h_{zm})\xi^m - (id_{z\alpha}-id^h_{z\alpha})\lambda^\alpha - (\partial\theta^\alpha - \partial\theta^{h\,\alpha})\chi_\alpha + \\
&+i\beta_{zm}(\lambda\gamma^m\lambda) + 2(\kappa_z\gamma_m\lambda)\xi^m - \pi_m\xi'^m - \pi_{\underline{\alpha}}\chi'_\alpha + b'_{zz}\gamma'^z + b_{zz}\gamma^z + \\
&+c^z\left(T_{zz}+\frac{1}{2}T_{zz}^{\mathrm{top}}\right) + \gamma^z\left(B_{zz}+\frac{1}{2}B_{zz}^{\mathrm{top}}\right) \qquad (5.11)
\end{aligned}
$$

with

$$
\begin{aligned}
T_{zz} = &-\frac{1}{2}\Pi_{zm}\Pi_z^m + d_{z\alpha}\partial\theta^\alpha + \frac{1}{2}\Pi^h_{zm}\Pi_z^{h\,m} - d^h_{z\alpha}\partial\theta^{h\,\alpha} + \beta_{zm}\partial\xi^m + \omega_{z\alpha}\partial\lambda^\alpha + \kappa_z^\alpha\partial\chi_\alpha + \\
&+\beta'_{zm}\partial\xi'^m + \kappa'^\alpha_z\partial\chi'_\alpha + \pi_{z\mathfrak{a}}\partial\varphi^{\mathfrak{a}} + 2\beta'_{zz}\partial\gamma'^z + \partial\beta'_{zz}\gamma'^z + 2b'_{zz}\partial c'^z + \partial b'_{zz}c'^z \qquad (5.12)\\
B_{zz} = &-\frac{1}{2}(\Pi_{zm}+\Pi^h_{zm})\beta^m + \frac{i}{2}(id_{z\alpha}+id^h_{z\alpha})\kappa_z^\alpha - \frac{i}{2}(\partial\theta^\alpha+\partial\theta^{h\,\alpha})\omega_{z\alpha} + \\
&+\beta'_{zm}\partial\varphi^m + \kappa'^\alpha_z\partial\varphi^{\underline{\alpha}} - 2\beta'_{zz}\partial c'^z - c'^z\partial\beta'_{zz} - b'_{zz} - \frac{1}{2}c'^zF_{zzz} - \frac{1}{2}\gamma'^z\Phi_{zzz}\,. \qquad (5.13)
\end{aligned}
$$

The final energy-momentum tensor, composite B-field and ghost current are

$$
\begin{aligned}
\mathbf{T}_{zz} &= T_{zz} + T^{\mathrm{top}} = T_{zz} + 2\beta_{zz}\partial\gamma^z + \partial\beta_{zz}\gamma^z + 2b_{zz}\partial c^z + \partial b_{zz}c^z && (5.14)\\
\mathbf{B}_{zz} &= B_{zz} + B^{\mathrm{top}} = B_{zz} - 2\partial c^z\,\beta_{zz} - c^z\partial\beta_{zz} && (5.15)\\
\mathbf{j}_z^{\mathrm{gh}} &= \beta_{zm}\xi^m + \omega_{z\alpha}\lambda^\alpha + \kappa_z^\alpha\chi_\alpha + \beta'_{zm}\xi'^m + \kappa'^\alpha_z\chi'_\alpha + \\
&\quad +b'_{zz}c'^z + 2\beta'_{zz}\gamma'^z + b_{zz}c^z + 2\beta_{zz}\gamma^z\,. && (5.16)
\end{aligned}
$$

Up to this point the above considerations were independent of the second BRST operator. The straightforward definition $\mathbf{Q}_c = [\mathbf{Q}, K]$ with the simple homotopy K that we used above, unfortunately, does not yield a nilpotent $\mathbf{Q}_c$ because of quadratic and cubic antighost terms in the generators F_{zzz} and Φ_{zzz} in eqs. (4.15) and (4.16), whose contribution to $\mathbf{Q}$ does not vanish in $\mathbf{Q}_c^2 = -\frac{1}{2}[\mathbf{Q}, [K, [K, \mathbf{Q}]]]$.

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6. Conclusions and outlook

Starting from the classical Green Schwarz action we have constructed the type-II version of the covariant superstring of Grassi, Policastro, Porrati and van Nieuwenhuizen. In the first part of the paper we were aiming at a transparent discussion of off-shell symmetries and their relation to the on-shell constraint algebras. The gauging of the WZNW model was performed in an unconventional way that was guided by the cancellation of the central terms in the constraint algebra. Circumventing the complications of the super group manifold approach we thus arrived at a free action with a nilpotent BRST charge in a simple and straightforward way.

The next step was the construction of the second BRST charge Q_c along the lines suggested by Grassi and van Nieuwenhuizen in [9]. Such a charge is easily constructed using a homotopy, which implies nilpotency due to the Jacobi identities. A modification of our charge may, however, be necessary, because the difference to Berkovits' BRST operator is not yet Q_c exact.

One problem of the Berkovits approach that could be solved in [7] by adding the topological quartet is the absence of diffeomorphism ghosts. With this quartet they also converted the Kazama algebra into a twisted $N = 2$ algebra. We observe, however, that one first has to obtain $N = 2$ with an additional quartet before diffeomorphisms (together with the accompanying fermionic symmetry) can be gauged.

Once the free action and the BRST charges that define the relative cohomology are constructed, it is clear that a number of tests concerning the viability of this proposal should be made. Another crucial question is the possible coupling of background fields, which again can be studied by cohomological techniques of deformation theory. It would be quite interesting to gauge the world sheet translations and their fermionic partner symmetry and thus obtain the topological quartet through a gauge fixing procedure of a diffeomorphism invariant theory on the world sheet.

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A. Conventions

The main differences between our conventions and those of [7] are:

$$p \to -p \tag{A.1}$$

$$d \to -d \tag{A.2}$$

$$
\begin{aligned}
(\beta_{zm},\omega_{z\alpha},\kappa_z^\alpha,b_{zz},\beta_{zz}) &\to (-\beta_{zm},-\omega_{z\alpha},-\kappa_z^\alpha,-b_{zz},-\beta_{zz}) && \text{(A.3)}\\
J_M &\neq (J_m,J_\alpha,J^\alpha), \quad J^M \neq (J^m, iJ^\alpha, -iJ^\alpha) && \text{(A.4)}\\
\text{but } J_M &= (J_m,J_\alpha,J_{\underline{\alpha}}) = (J_m,J_\alpha,-iJ^\alpha) && \text{(A.5)}\\
J^M &= (J^m,J^\alpha,J^{\underline{\alpha}}) = (J^m,J^\alpha,-iJ_\alpha) && \text{(A.6)}\\
\mathcal{H}^{MN} &\to (-)^N\mathcal{H}^{MN} && \text{(A.7)}\\
f^K{}_{MN} &\to -f^K{}_{MN}\,. && \text{(A.8)}
\end{aligned}
$$

All other conventions are practically the same to make comparison most simple. This includes a metric of the signature $(-,+)$ and the definition of the lightcone coordinates in the way

$$
\begin{aligned}
\sigma^- &\equiv \frac{1}{2}(\sigma^1-\sigma^0) && \overset{\sigma^0=-i\sigma^2}{\longrightarrow} && \frac{1}{2}(\sigma^1+i\sigma^2)\equiv z && \text{(A.9)}\\
\sigma^+ &\equiv \frac{1}{2}(\sigma^1+\sigma^0) && \overset{\sigma^0=-i\sigma^2}{\longrightarrow} && \frac{1}{2}(\sigma^1-i\sigma^2)\equiv \bar{z} && \text{(A.10)}\\
\partial\equiv\partial_- &= \partial_1-\partial_0 && \overset{\sigma^0=-i\sigma^2}{\longrightarrow} && \partial_1-i\partial_2=\partial_z\equiv\partial && \text{(A.11)}\\
\bar\partial\equiv\partial_+ &= \partial_1+\partial_0 && \overset{\sigma^0=-i\sigma^2}{\longrightarrow} && \partial_1+i\partial_2=\partial_{\bar z}\equiv\bar\partial && \text{(A.12)}\\
g_{-+} &= 2 = g_{z\bar z} && && && \text{(A.13)}\\
g^{-+} &= \frac{1}{2}g^{z\bar z}\,. && && && \text{(A.14)}
\end{aligned}
$$

The right hand side is the definition of the complex coordinates of the euclidean theory. We do not distinguish between ∂_- and ∂_z and call both of them ∂. The conformal map of the closed string worldsheet to the punctured complex plane has to look as follows

$$
z' = e^{-2iz} = e^{-i\sigma^1+\sigma^2}\,. \tag{A.15}
$$

We will not distinguish between z and z'. In OPE's, the variable z is the one of the punctured plane.

Wherever the string parameter α' is suppressed, it is set to

$$
\alpha' = 2\,. \tag{A.16}
$$

For the representation of the Gamma matrices, we refer to [6, p.14]. All what we need from the discussion in there is

$$
C\Gamma^m = \begin{pmatrix} \gamma^m_{\alpha\beta} & 0 \\ 0 & \gamma^{m\,\alpha\beta}\end{pmatrix}. \tag{A.17}
$$

With *symmetric* matrices $\gamma^m_{\alpha\beta}$ and $\gamma^{m\,\alpha\beta}$ which both obey (up to the position of the indices) a *Fierz identity* of the form

$$
\gamma_{m(\alpha\beta}\gamma^m_{\gamma)\delta} = 0\,. \tag{A.18}
$$

In addition the following equation holds

$$
\gamma^m_{\alpha\beta}\gamma^{n\,\beta\gamma} + \gamma^n_{\alpha\beta}\gamma^{m\,\beta\gamma} = -2\eta^{mn}\delta^\gamma_\alpha\,. \tag{A.19}
$$

(Here we disagree with the sign of [6, p.16]). For a Dirac spinor $\Psi = (\psi^\alpha, \varphi_\alpha)$, we thus have

$$\bar{\Psi}\Gamma^m\Psi = \psi^\alpha\gamma^m_{\alpha\beta}\psi^\beta + \varphi_\alpha\gamma^{m\,\alpha\beta}\varphi_\beta \equiv (\psi\gamma^m\psi) + (\varphi\gamma^m\varphi)\,. \tag{A.20}$$

We will consider Majorana-Weyl Femions only. θ^α e.g. is real and has 16 components. A hat $\hat{}$ on the index allows to treat type-IIA and type-IIB strings at the same time:

$$\hat{\theta}^{\hat{\alpha}} \equiv \begin{cases} \hat{\theta}_\alpha & \text{for type-IIA} \\ \hat{\theta}^\alpha & \text{for type-IIB}\,. \end{cases} \tag{A.21}$$

In the text we often call the hatted variables "antichiral" and the others "chiral". This refers to "right-moving" and "left-moving" and does not necessarily imply type IIA.

A.1 Minkowskian and euclidean lagrangian

The *minkowskian* free field action is

$$S = \frac{1}{2\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{-g}\underbrace{\left(-\frac{1}{2}\partial_\mu x^m\partial^\mu x_m + P^{\mu\nu}p_{\mu\alpha}\partial_\nu\theta^\alpha\right)}_{\equiv\mathcal{L}} \tag{A.22}$$

$$\mathrm{d}^2\sigma = \mathrm{d}\sigma^0\mathrm{d}\sigma^1 \tag{A.23}$$

$$P^{\mu\nu} \equiv g^{\mu\nu} + \varepsilon^{\mu\nu}\,, \qquad \eta_{\mu\nu} = \mathrm{diag}(-1,1) \tag{A.24}$$

$$\varepsilon^{\mu\nu} \equiv \frac{\epsilon^{\mu\nu}}{\sqrt{-g}}\,, \qquad \epsilon^{01} = 1 = -\epsilon^{10}\,. \tag{A.25}$$

In our calculations we consider coordinates σ^0 and σ^1 for which we have a flat worldsheet metric $\eta^{\mu\nu}$ and stick to this σ^0 as the canonical time. This implies that we do not transform the measure, when we consider the Lightcone coordinates. We denote by $\mathcal{L}$ not the complete lagrangian but only the inner (scalar) part

$$\mathcal{L} \equiv -\frac{1}{2}\partial_\mu x^m\partial^\mu x_m + P^{\mu\nu}p_{\mu\alpha}\partial_\nu\theta^\alpha\,. \tag{A.26}$$

So if we change to lightcone coordinates, we do not have to consider a factor of 2 coming from $\sqrt{-g}$. One can then just write an equality of the form

$$\mathcal{L} \equiv -\frac{1}{2}\partial_\mu x^m\partial^\mu x_m + P^{\mu\nu}p_{\mu\alpha}\partial_\nu\theta^\alpha \tag{A.27}$$

$$= -\frac{1}{2}\partial x^m\bar{\partial}x_m + p_{-\alpha}\bar{\partial}\theta^\alpha\,. \tag{A.28}$$

We arrive at the *euclidean* action by replacing the minkowskian by an euclidean metric, multiplying with an overall minus sign and redefining $\varepsilon^{\mu\nu}$ with an extra i.

$$S^E = -\frac{1}{2\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{g}\underbrace{\left(-\frac{1}{2}\partial_\mu x^m\partial^\mu x_m + P^{\mu\nu}p_{\mu\alpha}\partial_\nu\theta^\alpha\right)}_{\equiv\mathcal{L}^E} \tag{A.29}$$

$$\mathrm{d}^2\sigma = \mathrm{d}\sigma^2\mathrm{d}\sigma^1 \tag{A.30}$$

$$P^{\mu\nu} \equiv g^{\mu\nu} + \varepsilon^{\mu\nu} \tag{A.31}$$

$$\varepsilon^{\mu\nu} \equiv \frac{i\epsilon^{\mu\nu}}{\sqrt{g}}, \qquad \epsilon^{12} = 1 = -\epsilon^{21}. \tag{A.32}$$

For a flat metric we thus do not have to distinguish between minkowskian and euclidean lagrangian, so we will use the indices "$-$" and "z" synonymously.

Switching to *complex coordinates* in some sense undoes the Wick rotation as the determinant of the metric becomes negative. The measure transforms as follows

$$d^2 z \equiv \mathrm{d}z\mathrm{d}\bar{z} = \frac{1}{4}\left(\mathrm{d}\sigma^1 + i\mathrm{d}\sigma^2\right)\left(\mathrm{d}\sigma^1 - i\mathrm{d}\sigma^2\right) = \frac{i}{2}\mathrm{d}\sigma^2\mathrm{d}\sigma^1 = \frac{i}{2}\mathrm{d}^2\sigma \tag{A.33}$$

$$\mathrm{d}^2\sigma = -2i\mathrm{d}^2 z \tag{A.34}$$

$$S^E_{\mathrm{comp}} = \frac{i}{\pi\alpha'}\int \mathrm{d}^2 z \underbrace{\left(-\frac{1}{2}\partial x^m \bar{\partial} x_m + p_{z\alpha}\bar{\partial}\theta^\alpha\right)}_{\mathcal{L}^E_{\mathrm{comp}}} \tag{A.35}$$

$$\varepsilon^{z\bar{z}} = -\frac{1}{2} = \varepsilon^{-+} \tag{A.36}$$

$$\varepsilon_{z\bar{z}} = 2 = \varepsilon_{-+}. \tag{A.37}$$

In explicit calculations we do not specify whether we are in the euclidean or in the minkowskian case and thus just write

$$S = \int \mathcal{L} \tag{A.38}$$

where all the necessary prefactors and the measure are part of the $\int$-sign. As it looks nicer, we will write the index z whenever we are either in lightcone or in complex coordinates. We treat these coordinates in the same way, as already mentioned.

A.2 Superspace-conventions

Similar to [7], we use strict Southwest-Northeast conventions (NE for short) for capital indices, i.e. they are contracted in the following way:

$$T_M J^M = (-)^M J^M T_M. \tag{A.39}$$

In this convention it also makes sense to introduce two different Kronecker-deltas:

$$\delta^M_N \equiv \delta^M{}_N = (-)^{MN}\delta_N{}^M = (-)^M \delta_N{}^M \tag{A.40}$$

where the lefthand side is numerically equal to the usual Kronecker delta. However, we do not make this distinction for small indices, because they are always of definite grading

$$\delta^\beta_\alpha = \delta_\alpha{}^\beta = \delta^\beta{}_\alpha. \tag{A.41}$$

Also the contraction direction (NE) is not important for the small indices. For matrices we define a *graded inverse* that yields the appropriate Kronecker delta. In the case of a metric the graded inverse gets the same symbol with different index positions:

$$\mathcal{H}_{MP}\mathcal{H}^{PN} = \delta_M{}^N = (-)^M \delta^N_M \tag{A.42}$$

$$(-)^P \mathcal{H}^{MP} \mathcal{H}_{PN} = \delta^M{}_N = \delta^N_M \,. \tag{A.43}$$

From the second line one sees how the graded inverse is numerically related to the ordinary inverse:

$$(\mathcal{H}^{-1})^{MP} = (-)^P \mathcal{H}^{MP} \,. \tag{A.44}$$

We denote the graded commutator always with the ordinary squared brackets:

$$[A, B] \equiv AB - (-)^{AB} BA \,. \tag{A.45}$$

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